# WIND IMAGING INTERFEROMETER

## *WINDII*

# ALGORITHM

# DESCRIPTION

~~Version~~

ISSUE ~~3.0~~ 3.0    OK.

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## Preface

The algorithms in this document are in part still under development. The majority of them have been tested and proven to work correctly in a mildly structured atmosphere, however, section 8.3 (algorithm A313), which derives the apparent quantities, requires further verification in the case of waves. Wave detection criteria have been included to flag images influenced by the presence of atmospheric waves.

The Characterization Data Base (CDB) contains instrument calibration data and derived values which are required by several of the algorithms described in this document. Atmospheric data are corrected using the CDB, therefore, any errors in the CDB values will translate to errors in the derived values from the SDPPS. It is important to consider the impact of systematic errors introduced by uncertainties in the CDB when using WINDII data.

The Algorithm Team is under the authority of the WINDII Science Team and has been given the responsibility to define and test the algorithms required for the Scientific Production Processing Software. The team members are:

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# INTRODUCTION

This document is a description of the algorithms needed to process the WINDII data. Processing is done on a daily basis and on raw data files that span one 24 hour period. The algorithms are set up to operate on measurements in sequence. One measurement is composed of a background image and 8 phase images. Frequent calibration measurements are also processed and used by the atmospheric measurements. Hence these data would be processed first and the results stored for later use.

A standard notation has been used to describe the image data. A given image is divided into two fields of view with rows and columns in each. This is indicated by subscripts *hij* with *h* indexing the field of view, *i* indexing the horizontal row and *j* indexing the vertical column. Each image in a measurement is indexed with *k*. Hence the intensity in field of view *h* at bin location $(i,j)$ and in image or phase step *k* is denoted as $I_{hijk}$. Each algorithm normally operates on the full image set of one measurement.

Instrumental effects are described primarily in the Characterization Data Base (CDB). A standard bin is used in the CDB which may not be the same size as a given measurement bin. In all cases the measurement bin is to be an integral multiple of CDB bins. This allows the CDB bins to be combined to provide the instrumental effects at the measurement binning. CDB bins are given index subscripts hpg where h = Fov, p = row and g = column.

*CDB bin noted P8*

Error estimates and error propagation are all taken to be first order and are based on the form:

For a function $x = f(u, v, w, ...)$ the variance is given by

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + \sigma_w^2 \left(\frac{\partial x}{\partial w}\right)^2 + ...$$

Deviations from these basic assumptions will be noted in the individual algorithms. However, if no difference is noted then it can be assumed that the basic relations hold.

# Table of Contents

Add apendix:

Appendix 1    – CDR description

Appendix 2    – SDPPS Algorithm Flow (of algorithms)

Appendix 3    – Quality flag settings (meaning)

# Part I
# A1-Telemetry Depacking

# CHAPTER 1

# A11 - Quality checks

The level 0 quality data is checked for bad data, telemetry drop outs, or missing data. Useable data do not exist during these times and thus no data can be catalogued under these conditions. An error report is generated in the log files.

When the quality is bad for the measurement header the meas. is rejected, but when the error occurs in the image data, a quality flag is set and the measurement is kept. Quality checking is set by a run time parameter.

Watchdog flag is checked if then saved C-CPARDEom parameter is set. The file is in the auxilary directory.

# CHAPTER 2
# A12 - Depack data

The level 0 telemetry data from WINDII is read and the header information is used to depack, or decommutate the different types of measurements. For observational data, three files are created with atmospheric and background measurements in one, frequent phase measurements in another, and frequent dark current measurements in the third. A measurement log file is created giving a detailed list of all the measurements. The data files are used by subsequent steps. (include file structures!!)

All meas are numbered with an ID number use info from detailed design. as given in the following table.

| Type | ID number. |
|---|---|
| dc. | 10,000 |
| phase | 20,000 |
| Bkgnd | 30,000 |
| meas. | 40,000 |
| OH filt 5 corrected | 50,000 |

# CHAPTER 3
# A13 - Catalogue data

The depacking function also separates the infrequent calibration data, memory dump data, and engineering data. These data are catalogued and are not processed any further by the SDPPS. Also a time grid which matches the current day's data, is created for later use. The creation of these files is reported in the log files.

( in auxilary directory )

# Part II
# A2 - Data Calibration

# CHAPTER 4

# A21 - Remove instrumental effects

## 4.1 A211 - Subtract dark current

### 4.1.1 A2111: Initialize bin flags

**FUNCTIONAL DESCRIPTION:**

The bad bin map in the Characterization Data Base (CDB) is searched and bins in the measurement are flagged with a '0' value if any bad bins are present. Otherwise the flag is initialized to a '1' value indicating a good bin.

**INPUT:**

The input parameters are:

- $Ntop$ - CDB bin number on vertical axis of the first bin of the window.
- $Nbot$ - CDB bin number on vertical axis of the last bin of the window.
- $Nrht$ - CDB bin number on horizontal axis of the first bin of the window.
- $Nlft$ - CDB bin number on horizontal axis of the last bin of the window.
- $N_i$ - Number of measurement bins in the vertical in the window.
- $N_j$ - Number of measurement bins in the horizontal in the window.
- $N_k$ - Number of images in the observation, calibration or background measurement.
- $BBmap_{hpq}$ - Bad Bin map from the CDB. (0 for bad, 1 for good)

*[handwritten: ↗ Max Bin intensity (CDB)]*

**OUTPUT:**

*[handwritten: $I_{Max}$]*

The output parameters are:

- $Flags_{hijk}$ - Flags mapping the good and bad measurement bins.

### HYPOTHESIS OF APPLICABILITY:

The CDB window is always the full operating area of the detector and the CDB bins are always 5 pixels in the horizontal by 1 pixel in the vertical. Each measurement bin is always an integral number of CDB bins. The measurement window may be located anywhere on the detector area. The top, bottom, left and right sides of the measurement window are defined in terms of the CDB bins.

### CRITERIA OF APPLICABILITY:

Applied to all measurement and calibration images.

### TRANSFORMATION EQUATION:

All flags and counters are integer values and so the division operations which define $Np$, $Nq$, $i$ and $j$ are true integer divisions with no remainder. The flag value indicating a bad bin is always '0' and a good bin is '1'.

$$Flags_{hijk} = 1 \qquad \forall \ h, i, j \text{ and } k. \tag{EQ 4.1}$$

$$N_p = \frac{N_{bot} - N_{top} + 1}{N_i} \tag{EQ 4.2}$$

$$N_q = \frac{N_{lft} - N_{rht} + 1}{N_j} \tag{EQ 4.3}$$

$$Flags_{hijk} = Flags_{hijk} \times BBmap_{hpq} \qquad \forall \ h \text{ and } k.$$

$$\text{for } p = Ntop \text{ to } Nbot \text{ and } q = Nrht \text{ to } Nlft \tag{EQ 4.4}$$

where:

$$i = \left(\frac{p - Ntop}{N_p}\right) + 1 \tag{EQ 4.5}$$

$$j = \left(\frac{q - Nrht}{N_q}\right) + 1 \tag{EQ 4.6}$$

- look for saturation in image
- Set flags

#### 4.1.2 A2112: Dark current from frequent calibration

##### FUNCTIONAL DESCRIPTION:

A linear interpolation in time is made between the dark current measurements made during frequent calibrations before and after the measurement image that is to be corrected. The dark current monitor from the same frequent calibrations is also interpolated and the dark current mask is adjusted using the dark current monitor for the current image. The adjustment is made only if the difference between the image dark current monitor and the interpolated monitor value exceeds some predefined limit.

## INPUT:

The input parameters are:

- $I_{M_{hijk}}$ - measurement image bin values from depacking function.
- $DC_{hij}(t_1)$ - dark current map obtained at time $t_1$ during the last frequent calibration prior to the measurement image.
- $DC_{hij}(t_2)$ - dark current map obtained at $t_2$ in the first frequent calibration following the measurement image.
- $BB_{dcm_q}$ - Bad bin map for the dark current monitor (from CDB).
- $dcm_{qk}$ - four values of dark current monitor for each measurement image. ($q = 1$ to $4$ and $k = 1$ to $Nk$)
- $dcm(t_1)_q$ - four values of the dark current monitor for the last frequent calibration before the measurement.
- $dcm(t_2)_q$ - four values of the dark current monitor for the first frequent calibration following the measurement.
- $t_1$ - time of the first frequent calibration.
- $t_2$ - time of the second frequent calibration.
- $t_k$ - time of the measurement image.
- $DTH$ - Dark current threshold level (from CDB).
- $a_{DC_{hpq}}$ - Dark current bias value or analog to digital converter bias value (from CDB). *from CDB I*
- $a_{dcm}$ - Dark current bias value for the dark current monitor pixels (from CDB).

## OUTPUT:

The output parameters are:

- $DCA_{hijk}$ - dark current map corresponding to the time of the measurement image adjusted for the dark current monitor (index $k$ indicates the phase image).
- $I_{MC_{hijk}}$ - measurement image corrected for dark current bias.

## HYPOTHESIS OF APPLICABILITY:

Assumes that the dark current varies linearly with time between frequent calibrations.

## CRITERIA OF APPLICABILITY:

Can be carried out only if the binning for the dark current is the same as that of the measurement image. Also assumes that frequent calibrations are available both before and after the measurement image. Where this is not so, a single dark current measurement is used and a quality flag is set. The procedure is critically dependent on the choice of threshold, initially set to $DTH = 3$ counts. This value can be adjusted with experience in orbit. If the process proves unnecessary, the threshold can simply be set to a high value. Normally frequent calibrations are done every ⑮ minutes. If no frequent calibration is available within ⑮ minutes then the ~~most recent calibration measurement correspond-ing to the atmospheric measurement is used~~ *more than $\Delta T_{CDB}$ from the calibration are rejected.*

*Catalogued after De model.*

*$\Delta T_{CDB}$.*

**TRANSFORMATION EQUATION:**

Combine the CDB dark current bias values to obtain average value for measurement bins. (See A2113 below for CDB bin conversion)

$$a_{DC_{hij}} = \text{average of } a_{DC_{hpq}} \quad \forall\, (p, q) \text{ in } (i, j) \tag{EQ 4.7}$$

Average the dark current monitor values accounting for bad bins.

$$\text{Let } N = \sum_{q=1}^{4} BB_{dcm_q} \tag{EQ 4.8}$$

$$dcm_k = \frac{1}{N} \sum_{q=1}^{4} dcm_{qk} BB_{dcm_q} \tag{EQ 4.9}$$

$$dcm(t_1) = \frac{1}{N} \sum_{q=1}^{4} dcm(t_1)_q BB_{dcm_q} \tag{EQ 4.10}$$

$$dcm(t_2) = \frac{1}{N} \sum_{q=1}^{4} dcm(t_2)_q BB_{dcm_q} \tag{EQ 4.11}$$

*for each Dark Current measurement, call A25 if it is requested, and dark current is catalogued.*

Calculate the interpolated dark current for each measurement bin. This applies to both the atmospheric measurements and the frequent phase measurements.

$$DC_{hijk} = DC_{hij}(t_1) + \frac{t_k - t_1}{t_2 - t_1} \times [DC_{hij}(t_2) - DC_{hij}(t_1)] \tag{EQ 4.12}$$

Calculate the interpolated dark current monitor value. This is applied to both the atmospheric and frequent phase measurements.

$$dcmi_k = dcm(t_1) + \frac{t_k - t_1}{t_2 - t_1} [dcm(t_2) - dcm(t_1)] \tag{EQ 4.13}$$

If $|dcmi_k - dcm_k| \geq DTH$ (counts) then the dark current map is corrected as follows:

$$DCA_{hijk} = (DC_{hijk} - a_{DC_{hij}}) \times \frac{dcm_k - a_{dcm}}{dcmi_k - a_{dcm}} \tag{EQ 4.14}$$

otherwise

*some but exposure different.*

$$DCA_{hijk} = DC_{hijk} - a_{DC_{hij}} \tag{EQ 4.15}$$

The value of $dcm_k$ will be much larger than the value of $dcmi_k$ in the case of the images of the frequent phase calibration. This is because the integration time of the frequent

*? Consider making DC = rate only so that you can use the exposure time to compute DC for phase and window type for case of*

phase calibration is much longer than the integration time of the dark current image. Therefore the difference in dark current due to the difference in integration time is reflected in the ratio in 4.14 and this corrects the dark current map to the appropriate value for the frequent phase.

Remove the dark current bias from the measurment data.

$$I_{MC_{hijk}} = I_{M_{hijk}} - a_{DC_{hij}}$$ **(EQ 4.16)**

### 4.1.3  A2113: CDB bin conversion

#### FUNCTIONAL DESCRIPTION:

The responsivity conversion factors and the filter transmittances for the CDB bins within the current specified measurement bin are averaged to produce the values for the current measurement binning.

*No filter transmittance for filter 1.*

#### INPUT:

The input parameters are:

- $Ntop$ - CDB bin number on vertical axis of the first bin of the window.
- $Nbot$ - CDB bin number on vertical axis of the last bin of the window.
- $Nrht$ - CDB bin number on horizontal axis of the first bin of the window.
- $Nlft$ - CDB bin number on horizontal axis of the last bin of the window.
- $N_i$ - Number of measurement bins or rows in the vertical in the window.
- $N_j$ - Number of measurement bins or columns in the horizontal in the window.
- $CR_{hpq}$ - CDB responsivity data (in counts/Rayleigh-sec) for the appropriate filter and aperture (open or closed), for the CDB binning.
- $CA_{hpq}$, $CB_{hpq}$ and $CC_{hpq}$ - CDB responsivity linearity correction factors, for the CDB binning. (these are 'white light' values, that is they are not wavelength dependent)
- $CT_{hpq}$ - CDB filter transmittance value for the emission line and filter of the measurement, for the CDB binning. Transmittance is normalized to unity at the filter peak.

#### OUTPUT:

The output parameters are:

- $MR_{hij}$ - responsivity conversion consistent with the binning of the current measurement image.
- $MA_{hij}$, $MB_{hij}$ and $MC_{hij}$ - responsivity linearity correction factors consistent with the binning of the current measurement image
- $MT_{hij}$ - filter transmittance consistent with the binning of the current measurement image.

#### HYPOTHESIS OF APPLICABILITY:

Assumes that the responsivity and transmittance of a bin can be obtained from the responsivities and transmittances of the pixels within it simply by averaging.

#### CRITERIA OF APPLICABILITY:

Generally applicable.

#### TRANSFORMATION EQUATION:

The indecies are all integers and so the division operations which define $i$ and $j$ are true integer divisions with no remainder. Index $p$ increases from $Ntop$ to $Nbot$ and index $q$ from $Nrht$ to $Nlft$.

$$\text{Let } N_p = \frac{Nbot - Ntop + 1}{N_i} \text{ and } N_q = \frac{Nlft - Nrht + 1}{N_j} \qquad \text{(EQ 4.17)}$$

$$\text{then } N_{BC} = N_p \times N_q$$

$$MR_{hij} = \sum_p \sum_q CR_{hpq} \quad \forall\, (p, q) \text{ in } (i, j). \qquad \text{(EQ 4.18)}$$

where

$$i = \left( \frac{p - Ntop}{N_p} \right) + 1 \qquad \text{(EQ 4.19)}$$

$$j = \left( \frac{q - Nrht}{N_q} \right) + 1 \qquad \text{(EQ 4.20)}$$

The same equation applies to $MA_{hij}$, $MB_{hij}$ and $MC_{hij}$, using $\frac{1}{N_{BC}}$ to average over the measurement bin. Similarly for the transmittances:

$$MT_{hij} = \frac{1}{N_{BC}} \sum_p \sum_q CT_{hpq} \quad \forall\, (p, q) \text{ in } (i, j) \qquad \text{(EQ 4.21)}$$

with $i$ and $j$ as above.

### 4.1.4  A2114: Responsivity linearity

#### FUNCTIONAL DESCRIPTION:

The dimensionless correction factor is provided by a third order polynomial in the signal level.

#### INPUT:

The input parameters are:

- $I_{MC_{hijk}}$ - Dark current bias corrected signal level (in counts)

- $DCA_{hijk}$ - Dark current for measurement bins.

- $MA_{hij}$, $MB_{hij}$ and $MC_{hij}$ - values computed in A2113 for the linearity polynomial given below.

**OUTPUT:**

The output parameters are:

- $L_{I_{hijk}}$ - linearity correction factor for intensity.

- $L_{D_{hijk}}$ - linearity correction factor for dark current. itemize

**HYPOTHESIS OF APPLICABILITY:**

Assumes that the signal light level corresponding to a bin can be obtained from the responsivity that characterizes that bin, multiplied by a dimensionless linearity polynomial.

**CRITERIA OF APPLICABILITY:**

If the CCD proves to be accurately linear, this processing step will be bypassed. Or it may turn out that A, B and C may be represented by constants for the whole CCD. Switches should be provided to allow for these possibilities. Alternatively it may be necessary to apply the linearity correction only to signals in a given range. These questions will be answered after the instrument is characterized.

**TRANSFORMATION EQUATION:**

$$L_{I_{hijk}} = 1 + MA_{hij} \times \left[I_{MC_{hijk}}\right] + MB_{hij} \times \left[I_{MC_{hijk}}\right]^2 + MC_{hij} \times \left[I_{MC_{hijk}}\right]^3 \qquad \textbf{(EQ 4.22)}$$

$$L_{D_{hijk}} = 1 + MA_{hij} \times [DCA_{hijk}] + MB_{hij} \times [DCA_{hijk}]^2 + MC_{hij} \times [DCA_{hijk}]^3 \qquad \textbf{(EQ 4.23)}$$

### 4.1.5 A2115: Subtract dark current

**FUNCTIONAL DESCRIPTION:**

The dark current obtained from A2112 is subtracted from the measurement image.

**INPUT:**

The input parameters are:

- $DCA_{hijk}$ - dark current map obtained from function A2112.

- $I_{MC_{hijk}}$ - measurement image values from function A2112.

- $L_{I_{hijk}}$ and $L_{D_{hijk}}$ - linearity factors from A2114.

- *Convert* - electron to count conversion factor (from CDB).

- *DRE* - estimated digitization + readout + electronic noise contribution to variance (from CDB).

**OUTPUT:**

The output parameters are:

- $I_{DC_{hijk}}$ - measurement image corrected for dark current.

- $\sigma^2_{DC_{hijk}}$ - Initial error variance estimate (incounts).

## HYPOTHESIS OF APPLICABILITY:
As in A2111 and A2112.

## CRITERIA OF APPLICABILITY:
Requires that a dark current image was obtained from A2112.

## TRANSFORMATION EQUATION:
Before subtracting the dark current check to make sure that the frequent dark current matches the atmospheric measurement in terms of binning and exposure time.

$$I_{DC_{hijk}} = \frac{I_{MC_{hijk}}}{L_{I_{hijk}}} - \frac{DCA_{hijk}}{L_{D_{hijk}}} \qquad \text{(EQ 4.24)}$$

$$\sigma^2_{DC_{hijk}} = \frac{\left[I_{MC_{hijk}} + DCA_{hijk}\right]}{Convert} + DRE \qquad \text{(EQ 4.25)}$$

### 4.1.6 A2116: Set bin flag indicating clipping

#### FUNCTIONAL DESCRIPTION:
The dark current corrected data are compared with the noise level of the dark image to determine whether the phase waveform has been clipped. Clipping occurs when the intensity of the observed line is weak and the measured data points are close to the noise level. Certain points on the waveform are then lost in the noise and are not valid data. Also since the dark current has been subtracted it is possible for the corrected signal to be negative. This function is intended to check for these bad data points. If clipping is detected a flag is set which is used in the apparent quantity calculation to reject affected data points.

#### INPUT:
The input parameters are:

- $Flags_{hijk}$ - flags indicating the goodness of the bin data (see A2111).

- $I_{DC_{hijk}}$ - measurement image values corrected for dark current.

- $\sigma_h$ - average standard deviation of dark current (from CDB).

#### OUTPUT:
The output parameters are:

- $Flags_{hijk}$ - quality flags which indicate the goodness of the bin data.

**HYPOTHESIS OF APPLICABILITY:**

When low light levels are measured the phase waveform may be clipped for certain phase steps. If the data are not rejected from the calculation of the apparent quantities then large distortions will occur. The clipping results from the subtraction of a dark current from a signal of comparable magnitude. It is not possible to estimate, even for negative values, the actual modulation depth. Hence it is better to flag the data and not use them in the fit.

**CRITERIA OF APPLICABILITY:**

Applied to all images.

**TRANSFORMATION EQUATION:**

Each bin is checked against $\sigma_h$ as the indicator for clipping.

$$\text{If } I_{DC_{hijk}} \leq \sigma_h \text{ then } flags_{hijk} = 0 \qquad \textbf{(EQ 4.26)}$$

Note:

The variance of the dark current in each field of view stored in the CDB is computed from dark current images as follows. It is not expected that this calculation is needed for each dark current image as part of the SDPPS. If a representative value stored in the CDB proves to be unreliable then this calculation will need to be done for each different window and bin combination. The window and bin size will change with each filter group. The following is the definition of $\sigma_h$.

$$N = N_i \times N_j \qquad \textbf{(EQ 4.27)}$$

$$\overline{DCA_h} = \frac{1}{N} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} DC_{hij} \qquad \textbf{(EQ 4.28)}$$

$$\sigma_h^2 = \frac{1}{N-1} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} (DC_{hij} - \overline{DCA_h})^2 \qquad \textbf{(EQ 4.29)}$$

where $DC_{hij}$ is the dark current image.

## 4.2   A212 - Convert from counts to rayleighs

### 4.2.1   A2121: Convert counts to rayleighs

**FUNCTIONAL DESCRIPTION:**

The signal level in rayleighs is computed from the signal level in counts, knowing the exposure time, the responsivity value and the linearity factor.

**INPUTS:**

The input parameters are:

- $I_{DC_{hijk}}$ - Dark current corrected signal level (in counts) from A2115.

*(handwritten: Saved in Scratch File)*

- $\sigma^2_{DC_{hijk}}$ - Initial error variance estimate (in counts) from A2115.

- $t_{exp}$ - exposure time in seconds.

- $MR_{hij}$ - responsivities for the measurement bins, from A2113. (in counts/rayleigh)

### OUTPUT:

The output parameters are:

- $I_{CR_{hijk}}$ - signal level (in rayleighs).

- $\sigma^2_{CR_{hijk}}$ - Initial error variance estimate of signal level (in rayleighs).

### HYPOTHESIS OF APPLICABILITY:

Generally applicable.

### CRITERIA OF APPLICABILITY:

This process is applied to all observation images.

### TRANSFORMATION EQUATION:

$$I_{CR_{hijk}} = \frac{I_{DC_{hijk}}}{t_{exp} \times MR_{hij}} \qquad \text{(EQ 4.30)}$$

$$\sigma^2_{CR_{hijk}} = \frac{\sigma^2_{DC_{hijk}}}{(t_{exp} \times MR_{hij})^2} \qquad \text{(EQ 4.31)}$$

**CHAPTER 5**

# A22 - Remove observatory effect

## 5.1 A221 - Determine altitude of the bins

### 5.1.1 A2211: Determine ECI coordinates of line of sight

**FUNCTIONAL DESCRIPTION:**

This transformation gives as a function of bin location in the CCD the Earth Centred Inertial (ECI) coordinates of its corresponding line of sight. The OA_SAT_ATT Orbit/ Attitude service routine is used in this function.

**INPUT:**

The input parameters are:

- See functions A22111 to A22114.

**OUTPUT:**

The output parameters are:

- See functions A22111 to A22114.

HYPOTHESIS OF APPLICABILITY:

CRITERIA OF APPLICABILITY:

*Applies to atmospheric and background measurement not frequent calibration*

TRANSFORMATION EQUATION:

### 5.1.1.1 A22111: Determine bin centroid in FOV frame

FUNCTIONAL DESCRIPTION:

The bin centroid is calculated for each bin in the FOV in units of CCD pixels. The location of the upper window in the split filter (5) is determined in absolute location on the CCD and then is transformed to the correct altitude range in the next algorithm, A22112.

INPUT:

The input parameters are:

- $NVB$ - Number of pixels in the vertcal column in measurement bin.
- $NHB$ - Number of pixels in the horizontal in measurement bin.
- $NVW$ - Number of pixels in the veritcal column in the window.
- $NHW$ - Number of pixels in the horizontal in the window.
- $NVfov$ - Number of pixels in the vertical column in the field of view. (from the CDB)
- $NHfov$ - Number of pixels in the horizontal in the field of view. (from the CDB)
- $NVO$ - Number of pixels in the veritcal window offset.
- $NHO$ - Number of pixels in the horizontal window offset.
- $NWS$ - Number of pixels in the window separation.
- $N_i$ - Number of measurement bins in the vertical window.
- $N_j$ - Number of measurement bins in the horizontal.
- $Ntop$ - Pixel row, from the top of the CCD, of the top of the measurement window.

OUTPUT:

The output parameters are:

- $XC_{hij}$ - Bin centroid horizontal coordinate.
- $YC_{hij}$ - Bin centroid veritcal coordinate.

HYPOTHESIS OF APPLICABILITY:

Assumes all the window and bin information is available from the measurement header. Both fields of view have the same window dimensions and there is no change in window dimension during the time of one measurement. If there is a split window then both windows in each FOV have the same bin dimensions and window dimensions. *but may have offset.*

CRITERIA OF APPLICABILITY:

The split field filter 5 produces two windows in each field of view. The upper window is the background and the lower is the OH measurement. This routine should be called once for each of the upper and lower windows to produce two sets of $XC$ and $YC$. The

depacking function separates out the background and measurements and so treats filter 5A and 5B as two different images. The *Ntop* parameter is computed for each of 5A and 5B windows.

## TRANSFORMATION EQUATION:

Check that $N_i = \dfrac{NVW}{NVB}$ and that $N_j = \dfrac{NHW}{NHB}$. Also if there is a split window then the window separation *NWS* must be accounted for in the calculation of the bin *Ntop*. (See figure 5.1)

Each field of view is defined separately as follows:

$$XC_{1ij} = (j - 0.5) \times NHB + NHfov - NHO - NHW \qquad \textbf{(EQ 5.1)}$$

$$YC_{1ij} = (i - 0.5) \times NVB + Ntop - 1 \qquad \textbf{(EQ 5.2)}$$

$$XC_{2ij} = (j - 0.5) \times NHB + NHO \qquad \textbf{(EQ 5.3)}$$

$$YC_{2ij} = (i - 0.5) \times NVB + Ntop - 1 \qquad \textbf{(EQ 5.4)}$$

The value of *Ntop*, as computed in the depacking function, is:

For the normal case (including filter 5A):

$Ntop = NVccd - NVO - NVW + 1$

For the case of filter 5B:

$Ntop = NVccd - NVO - 2NVW - NWS + 1$

where *NVccd* = 256 is the total number of pixels in the CCD in the vertical.

**Figure 5.1:**      Bin and window location in the two fields of view (FOV) as seen from the image side.



Bin and pixel numbers are ordered in the same way as (i,j) pairs. C1 and C2 are the boresight locations in each FOV and OA as the axis of the imaging optics.

The boresights and optical axis are located at pixel coordinates, C1(255,80), C2(255,80) in each FOV and OA(133,160) if FOV2 or OA(133,0) in FOV1.

#### 5.1.1.2   A22112: Determine coordinates of line of sight in IPF

**FUNCTIONAL DESCRIPTION:**

Calculate the line of sight passing through a given point in the Instrument Pointing Frame (IPF) using the instrument angular field of view. This function uses the optical distortion measured in the instrument characterization to correct the flat field approximation for the position of a pixel in the reference frame. The image plane is first assumed to be flat with no distortion and the (X,Y) position of a given bin is transformed to angular position. The relative increase or decrease in angular position due to distortion is next calculated. The flat field angles may then be adjusted and the IPF directions calculated. Filter #5 is split using a wedge prism. This deflects the image to the upper half of the CCD detector and adds its own distortion. Hence this special case

is treated separately. In order to determine the correct pointing information the upper image in the split filter is transformed to the instrument pointing frame with its own transformation matrix.

## INPUT:

The input parameters are:

- $XC_{hij}$ - Bin centroid horizontal coordinate.

- $YC_{hij}$ - Bin centroid vertical coordinate.

- $\Delta\alpha$ - Angle,in radians, in horizontal of one FOV (nominal value is 0.06981 rad or 4°) (from the CDB)

- $\Delta\beta$ - Angle,in radians, in vertical of one FOV (nominal value is 0.10472 rad or 6°) (from the CDB)

- $NHfov$ - Total number of pixels in horizontal in FOV (from the CDB)

- $NVfov$ - Total number of pixels in vertical in FOV (from the CDB)

- $XOA_h, YOA_h$ - The (X,Y) location, in pixels, of the imaging optical axis for each field of view. (from the CDB)

- $a_{dis}, b_{dis}, c_{dis}$ - Coefficients from CDB to describe optical distortion. (referenced to the imaging optical axis)

- $aw_{dis}, bw_{dis}, cw_{dis}$ - Coefficients from CDB to describe optical distortion with wedge in filter 5. (referenced to the imaging optical axis)

- $\delta\alpha_h, \delta\beta_h, \delta5_h, \delta\beta5_h$ - Offset angles, from the CDB, to translate windows in each fov to the optical axis so that rotation due to prism missalignment may be corrected. There is a unique set used to translate filter 5B windows.

- $BS_h$ - Transform matrix for rotating bin position into IPF for each boresight in the normal case. (These are 2X2 matrices)

- $OA\alpha_h, OA\beta_h$ - Offset angles needed to translate the window or bin centroid from the optical axis reference frame to the boresight reference frame after distortion and rotation corrections.

## OUTPUT:

The output parameters are:

- $UIPF_{hij}(c)$ - Array of direction cosines of the line of sight of the bin centroids.

## HYPOTHESIS OF APPLICABILITY:

The angle subtended by a pixel is defined by the total angular field of view divided by the number of pixels in the field of view. The angle measure is radians. The angular FOV and the pixel dimensions for each FOV are assumed to be available from the CDB. It is assumed that the optical distortion is symmetric about the optical axis of the imaging optics. Hence the distortion correction is first made in a reference frame attached to the imaging optical axis at point $(XOA, YOA)$. The tilt of the Michelson may introduce some small asymmetry, however, the characterization of the instrument is required to verify this. Until this is done the symmetrical assumption will be used. Since the angles

used are referenced to the imaging optical axis the parameters $\Delta\alpha$, $\Delta\beta$, $a_{dis}$, $b_{dis}$, $c_{dis}$, $aw_{dis}$, $bw_{dis}$ and $cw_{dis}$ are independent of field of view.

### CRITERIA OF APPLICABILITY:

This function accounts for optical distortion within the field of view. This can be measured on the ground before flight, however, it cannot be measured directly in flight. Consequently once in orbit the two functions A22112 and A22113 cannot easily be separated. Any on orbit alignment measurements will measure the product of A22112 and A22113. However, if we assume that the optical distortion does not change after launch then a shift in the instrument pointing frame will cause a change only in $F_h$ (A22113). Hence we can adjust $F_h$ and not change A22112 parameters. This also has the advantage that only two transformation matrices need be saved in the CDB rather than one for each CDB bin.

Note that the definition of *NVfov* and $\Delta\beta$ must be consistent and that the pixel coordinates used for *YOA* and *YC* must be the same. This will avoid any problems with the difference between *NVccd* = 256 and *NVfov* = 240.

### TRANSFORMATION EQUATION:

See the definition of the UARS and instrument reference frames for definition of the angles. The distortion parameters calculate a relative increase or decrease in the angular position of a pixel based on the angular distance from the imaging optical axis.

$$\alpha o_{hij} = \left(\frac{XOA_h - XC_{hij}}{NHfov}\right) \times \Delta\alpha \qquad \textbf{(EQ 5.5)}$$

$$\beta o_{hij} = \left(\frac{YOA_h - YC_{hij}}{NVfov}\right) \times \Delta\beta \qquad \textbf{(EQ 5.6)}$$

Use the small angle approximation to calculate the angular distance from the imaging optical axis $(XOA, YOA)$,

$$\gamma o_{hij} = \sqrt{\alpha o_{hij}^2 + \beta o_{hij}^2} \qquad \textbf{(EQ 5.7)}$$

**If** normal case (not filter 5) **Then**

$$DIS_{hij} = a_{dis} + b_{dis}(\gamma o_{hij}) + c_{dis}(\gamma o_{hij})^2 \qquad \textbf{(EQ 5.8)}$$

**Else** (upper window of split filter 5)

$$DIS_{hij} = aw_{dis} + bw_{dis}(\gamma o_{hij}) + cw_{dis}(\gamma o_{hij})^2 \qquad \textbf{(EQ 5.9)}$$

Correct the flat field estimates of the angular position of the bin centroid for optical distortion.

$$\alpha_{hij} = DIS_{hij} \times \alpha o_{hij} \qquad \textbf{(EQ 5.10)}$$

$$\beta_{hij} = DIS_{hij} \times \beta o_{hij} \qquad \text{(EQ 5.11)}$$

Translate each FOV window to the imaging optical axis so that the windows may be rotated to account for any missalingment due to the prism edge.

**If** normal case (not filter 5) **Then**

$$\alpha 1_{hij} = \alpha_{hij} + \delta\alpha_h \qquad \text{(EQ 5.12)}$$

$$\beta 1_{hij} = \beta_{hij} + \delta\beta_h \qquad \text{(EQ 5.13)}$$

**Else** (upper window of split filter 5)

$$\alpha 1_{hij} = \alpha_{hij} + \delta\alpha 5_h \qquad \text{(EQ 5.14)}$$

$$\beta 1_{hij} = \beta_{hij} + \delta\beta 5_h \qquad \text{(EQ 5.15)}$$

Rotate the windows by angle $\theta_h$ using a single rotation matrix for each field of view.

$$\begin{bmatrix} \alpha 2_{hij} \\ \beta 2_{hij} \end{bmatrix} = BS_h \times \begin{bmatrix} \alpha 1_{hij} \\ \beta 1_{hij} \end{bmatrix} \qquad \text{(EQ 5.16)}$$

where the 2x2 matrix, $BS_h$ has to form:

$$BS_h = \begin{bmatrix} \cos\theta_h & -\sin\theta_h \\ \sin\theta_h & -\cos\theta_h \end{bmatrix} \qquad \text{(EQ 5.17)}$$

Now the location of the bin centroid has been corrected for distortion and rotation in the imaging optical axis reference frame. The last step is to translate to the boresight reference frame. This is done using CDB parameters as follows.

$$\alpha 3_{hij} = OA\alpha_h - \alpha 2_{hij} \qquad \text{(EQ 5.18)}$$

$$\beta 3_{hij} = OA\beta_h + \beta 2_{hij} \qquad \text{(EQ 5.19)}$$

The bin centroids have been located in the boresight reference frame so now the direction cosines may be computed.

$$UIPF_{hij}(z) = \cos\left(\sqrt{\alpha 3_{hij}^2 + \beta 3_{hij}^2}\right) \qquad \text{(EQ 5.20)}$$

$$UIPF_{hij}(x) = UIPF_{hij}(z) \times \tan(\beta 3_{hij}) \qquad \text{(EQ 5.21)}$$

$$UIPF_{hij}(y) = UIPF_{hij}(z) \times \tan(\alpha 3_{hij}) \qquad \text{(EQ 5.22)}$$

### 5.1.1.3   A22113: Determine OMF coordinates

#### FUNCTIONAL DESCRIPTION:

The vector in the Instrument Pointing Frame (IPF) is transformed to the vector in the Operational MACS Frame (OMF) according to the CDB transformation matrix $F_h$.

*[handwritten: IPF to OMF]*

#### INPUT:

The input parameters are:

- $UIPF_{hij}(c)$ - Array of direction cosines in the IPF.
- ~~$F_h$ - CDB transformation matrix from IPF to OMF.~~

#### OUTPUT:

The output parameters are:

- $VOMF_{hij}(c)$ - Direction cosine array in OMF coordinates.

#### HYPOTHESIS OF APPLICABILITY:

Two transformation matrices are defined in the CDB, one for each field of view. Each matrix $F_h$ is a 3X3 matrix with the general form:

*[handwritten: ← more space between columns →]*

$$F_h = T \times \begin{bmatrix} \cos\alpha_h\sin\beta_h\cos\gamma_h - \sin\alpha_h\sin\gamma_h & -\cos\alpha_h\sin\beta_h\sin\gamma_h - \sin\alpha_h\cos\gamma_h & \cos\alpha_h\cos\beta_h \\ \sin\alpha_h\sin\beta_h\cos\gamma_h + \cos\alpha_h\sin\gamma_h & -\sin\alpha_h\sin\beta_h\sin\gamma_h + \cos\alpha_h\cos\gamma_h & \sin\alpha_h\cos\beta_h \\ -\cos\beta_h\cos\gamma_h & \cos\beta_h\sin\gamma_h & \sin\beta_h \end{bmatrix}$$

where index $h$ indicates the field of view, $\alpha_h$ is the elevation angle, $\beta_h$ is the azimuth angle, $\gamma_h$ is the skew angle and T is the missalignment matrix (3x3) transforming the Instrument Reference Frame (IRF) to the Operational Macs Frame (OMF). *[handwritten: There is one value in the CDB for each yaw period.]*

#### CRITERIA OF APPLICABILITY:
None.

#### TRANSFORMATION EQUATION:

$$VOMF_{hij} = F_h \times UIPF_{hij} \qquad \text{(EQ 5.23)}$$

### 5.1.1.4   A22114: Determine coordinates in the ECI frame

#### FUNCTIONAL DESCRIPTION:

This transformation gives the Earth Centred Inertial (ECI) coordinates of the line of sight. The OA_SAT_ATT Orbit/Attitude service routine is used to obtain the transformation from Operational MACS frame (OMF) coordinates to ECI coordinates.

#### INPUT:

The input parameters are:

- $UT_k$ - Universal time of the measurement for each image.

*[handwritten margin notes:]*

*• $\alpha_h^y$ — Boresight elevation angle from CDB file C_IPF. One value per yaw period Y and for h.*

*• $\beta_h^y$ — Boresight azimuth angle ......*

*• $\gamma_h^y$ — Boresight skew angle .....*

*$T^Y$ — IRF misalignment transformation matrix from CDB file C_IPF. One value per yaw period Y.*

*change all $\alpha_h$ $\beta_h$ $\gamma_h$ to $\alpha_h^y$ $\beta_h^y$ $\gamma_h^y$*

- *TYPA* - Type of raw attitude data to be used (see OA_SAT_ATT).
- *VER* - Version of orbit data for calculation (see OA_SAT_ATT).
- $VOMF_{hij}(c)$ - Direction cosine array in OMF coordinates.

### OUTPUT:
The output parameters are:

- $VECI_{hijk}(c)$ - Direction cosine array in ECI coordintes for line of sight for each bin in each FOV.

### HYPOTHESIS OF APPLICABILITY:
The Orbit/Attitude service routine OA_SAT_ATT is called to obtain the transform from the OMF frame to the ECI frame. This transform may be used for all the bin line of sight vectors in the measurement image. Each phase image in a given measurement is taken at a different time, therefore, this routine needs to be called for each phase step.

### CRITERIA OF APPLICABILITY:
None.

### TRANSFORMATION EQUATION:
Call the OA_SAT_ATT service routine and obtain the 3X3 matrix *ABC* which is the transformation matrix from OMF to ECI for each phase image, *k*. Then,

$$VECI_{hijk} = (ABC)_k \times VOMF_{hij}$$

(EQ 5.24)

### 5.1.2   A2212: Determine tangent point altitude

#### FUNCTIONAL DESCRIPTION:
Orbit/Attitude service routines are used to obtain the tangent point altitude of each bin centroid. These altitudes are saved for each field of view and for each phase step. The velocity of the space craft projected along the line of sight and the velocity of the rotation of the earth on a latitude circle at the tangent point are also calculated for use in the extraction algorithms.

#### INPUT:
The input parameters are:

- $UT_k$ - Universal time of the measurement *for each image*
- $VECI_{hijk}(c)$ - Line of sight unit vector in ECI coordinates for each bin.
- *VER* - Version of orbit data for calculations.

#### OUTPUT:
The output parameters are:

- $Z_{hijk}$ - Geodetic tangent point altitude for each bin.

**HYPOTHESIS OF APPLICABILITY:**

In order to locate the measurement in the atmosphere, it is assumed that the bin centroid line of sight tangent point is representative of the measurement for that bin. Two Orbit_ Attitude service routines are called, OA_SAT_ORB and OA_LIMB_CALC.

**CRITERIA OF APPLICABILITY:**

None.

**TRANSFORMATION EQUATION:**

Call OA_SAT_ORB with $UT_k$ and *VER* set for the given image. *Psat*(3), the ECI position of the satellite, and *Vsat*(3), the ECI velocity vector of the space craft, are returned.

Call OA_LIMB_CALC with $UT_k$, *Psat* and $VECI_{hijk}$ for each bin, then:

$$Z_{hijk} = GEOD\_LAT \qquad \text{for each bin} \tag{EQ 5.25}$$

where *GEOD_ALT* is the geodetic altitude returned from the service routine.

### 5.1.3   A2213: Obtain reference tangent point for image

**FUNCTIONAL DESCRIPTION:**

Orbit/Attitude service routines are used to obtain the tangent point altitude, latitude and longitude for the reference column for the measurement. This reference column is used for the combination of the fields of view in A34.

**INPUT:**

The input parameters are:

- *NVB* - Number of pixels in the vertical in measurement bin.
- $N_i$, $N_j$ - Number of rows and columns in the measurement.
- $N_k$ - Number of phase images in the measurement (4 or 8).
- *NVW* - Number of pixels in the vertical in the window.
- *NHW* - Number of pixels in the horizontal in the window.
- *Ntop* - Number of pixels in the vertical to the top of the window. (from the CDB)
- *NHfov* - Number of pixels in the horizontal in the full field of view. (from the CDB)
- *NVO* - Number of pixels in the vertical window offset.(from bottom of image)
- *NHO* - Number of pixels in the horizontal window offset.
- $UT_k$ - Universal time of the measurement for each phase image.
- $VECI_{hijk}$ - Direction cosine array in ECI coordintes for line of sight for each bin in each FOV.
- $Z_{hijk}$ - Geodetic tangent point altitude for each bin.

**OUTPUT:**

The output parameters are:

- $ZR_{hi}$ - Reference geodetic tangent point altitude for each FOV.

- $\zeta R_{h i}$ - Reference longitude of tangent point.

- $\vartheta R_{h i}$ - Reference geodetic latitude of tangent point.

- $\Gamma R_{h i}$ - Reference angle of vector w.r.t. North.

- $\Delta Z R_{h}$ - Altitude overlap interval used in field combination.

- $\Delta \zeta R_{h}$ - Longitude overlap interval used in field combination.

- $\Delta \vartheta R_{h}$ - Latitude overlap interval used in field combination.

### HYPOTHESIS OF APPLICABILITY:

The middle image of a series of phase images that compose one measurement is used as the reference image. Image number 3 and image number 5 for the two cases of the 4 point and 8 point measurements respectively are used since the time assigned to the image will be the time at the start of the integration time. Since an average value for the apparent quantities is calculated in the extraction algorithms it is assumed that a single geo-reference point may be used to characterize the measurement. The point in the middle of each FOV and at the bin corresponding to the peak in the volume emission profile is chosen as the reference point. Since the volume emission profile is not known at this point a column of geo-reference data is computed and then scanned in A34 along with the volume emission rate profile to find the single reference bin. Previous functions as well as Orbit/Attitude service functions are required.

### CRITERIA OF APPLICABILITY:

A reference point is assigned to atmospheric measurements only and not to background measurements.

### TRANSFORMATION EQUATION:

First locate the pixels in the image reference column:

For each field of view ($h = 1$, $h = 2$) and with $k = 3$ for $N_k = 4$ or $k = 5$ for $N_k = 8$:

Set $j = \dfrac{N_j}{2}$ (integer division) and for each $i$ in the vertical column compute the bin centroids:

$$XC_1 = NHfov - NHO - \frac{NHW}{2} \qquad \text{(EQ 5.26)}$$

$$YC_{1 i} = (i - 0.5) \times NVB + Ntop - 1 \qquad \text{(EQ 5.27)}$$

$$XC_2 = \frac{NHW}{2} + NHO \qquad \text{(EQ 5.28)}$$

$$YC_{2 i} = (i - 0.5) \times NVB + Ntop - 1 \qquad \text{(EQ 5.29)}$$

Call functions A22112 through A22114 to obtain $VECI_{hi}(3)$ for the reference column. (This is different from the $VECI_{hijk}$ which are input to the routine. These inputs are used below.)

Call OA_SAT_ORB with UTk set for image $k = 3$ if 4 point or for image $k = 5$ if 8 point measurement. $Psat(3)$, the ECI position of the satellite, and $Vsat(3)$, the ECI velocity vector of the space craft, are returned.

Call OA_LIMB_CALC with $UT_k$, $Psat$ and $VECI_{hi}$ for each FOV, then:

$$ZR_{hi} = GEOD\_ALT_i \qquad \text{(EQ 5.30)}$$

$$\zeta R_{hi} = LONG_i \qquad \text{(EQ 5.31)}$$

$$\vartheta R_{hi} = GEOD\_LAT_i \qquad \text{(EQ 5.32)}$$

$$\Gamma R_{hi} = LOOK\_ANGLE_i \qquad \text{(EQ 5.33)}$$

where the right hand side of the above are returned from the service routine for each bin in the vertical column.

The overlap interval is determined from the spatial extent of the measurement based on the location of the first and last images in a measurement. The time of image $k = 1$ is $UT_1$ or $UT_{start} = UT_1$. The time of the last image is for $k = 4$ or $k = 8$ depending on whether a 4 or 8 point measurement has been taken. That is $UT_{end} = UT_4$ or $UT_8$.

Call OA_SAT_ORB with $UT_{start}$ and return $P_{sat_{start}}$.

Call OA_SAT_ORB with $UT_{end}$ and return $P_{sat_{end}}$.

With $i = \dfrac{2N_i}{3}$ (integer division), $j$ set as above, $start = 1$ and $end = 4$ or $8$ as before,

Call OA_LIMB_CALC with $UT_{start}$, $P_{sat_{start}}$ and $VECI_{h,i,j,start}$ and

Call OA_LIMB_CALC with $UT_{end}$, $P_{sat_{end}}$ and $VECI_{h,i,j,end}$

From the return values from the last two subroutine calls and from the input tangent altitudes calculate:

$$\Delta \zeta R_h = |LONG_{start} - LONG_{end}| \qquad \text{(EQ 5.34)}$$

$$\Delta \vartheta R_h = |GEOD\_LAT_{start} - GEOD\_LAT_{end}| \qquad \text{(EQ 5.35)}$$

$$\Delta ZR_h = \frac{|Z_{h,i,j,start} - Z_{h,i-1,j,start}| + |Z_{h,i,j,end} - Z_{h,i-1,j,end}|}{2} \qquad \text{(EQ 5.36)}$$

*(handwritten annotations in margin:)*

$\Delta \zeta R_h = |\min(LONG_{start,col\,1}, LONG_{start,col\,6}) - \max(LONG_{end,col\,1}, LONG_{end,col\,6})|$

$\Delta \theta R_h = |\min(GEOD\_LAT_{start,col\,1}, GEOD\_LAT_{start,col\,6}) - \max(GEOD\_LAT_{end,col\,1}, GEOD\_LAT_{end,col\,6})|$

Notes:

1. The point at $i = \dfrac{2N_i}{3}$ is used to obtain the overlap interval since this is approximately the expected region of the peak. The overlap interval does not change much from the bottom to the top of the reference column.

2. The Orbit Attitude service routines return values between 0° and 360° for the longitude. It is possible for the longitude to switch between 0 and 360 during the time of a measurement. The value of $\Delta \zeta R_h$ must be checked and corrected for this discontinuity.

3. The latitude changes very slowly at the poles and so the overlap interval will be very small. It may be necessary to define a minimum interval which would be derived from the other latitude intervals. This still needs to be confirmed.

## 5.2 A222 - Determine instrument phase

### 5.2.1 A2221: Derive projected velocities

**FUNCTIONAL DESCRIPTION:**
Orbit/Attitude service routines are used to obtain the velocity vector for the satellite. The velocity of the space craft projected along the line of sight and the velocity of the rotation of the earth on a latitude circle at the tangent point are also calculated for use in the extraction algorithms.

**INPUT:**
The input parameters are:

- $UT_k$ - Universal time of the measurement *of 3rd or 5th image ( k=3 or 8).*
- $VECI_{hij}(c)$ - Line of sight unit vector in ECI coordinates for each bin. *( k = 3 or 8).*

*$VECI_{hijk}(c)$*

- $VER$ - Version of orbit data for calculations.

**OUTPUT:**
The output parameters are:

- $vps_{hij}$ - Projected space craft velocity along line of sight. $(m\ s^{-1})$
- $vre_{hij}$ - Velocity of earth rotation at tangent point on a latitude circle for each bin. $(m\ s^{-1})$

**HYPOTHESIS OF APPLICABILITY:**
In order to locate the measurement in the atmosphere, it is assumed that the bin centroid line of sight tangent point is representative of the measurement for that bin. It is assumed that the projected velocities will not change from the first image to the last image of a measurement by a significant amount. Hence only the projected velocities for the *middle* image will be calculated. Two Orbit/Attitude service routines are called, OA_SAT_ORB and OA_LIMB_CALC.

## CRITERIA OF APPLICABILITY:
None.

## TRANSFORMATION EQUATION:
Call OA_SAT_ORB with *UT* and *VER* set for the given image. *Psat*(3), the ECI position of the satellite, and *Vsat*(3), the ECI velocity vector of the space craft, are returned. This is done for $k = 1$ only, hence the index on VECIhij1.

$k = 3$ or $5$

$$vps_{hij} = Vsat(\ ) \times VECI_{hijk} \quad \text{(vector scalar product)} \quad \text{(EQ 5.37)}$$

Call OA_LIMB_CALC with *UT*, *Psat* and *VECI*$_{hijk}$ for each bin, then:

The rotation of the earth induces a phase change comparable to the atmospheric wind phase. To account for this we must calculate the velocity of rotation at the tangent point along a latitude circle which is projected along the line of sight. From OA_LIMB_-CALC we have:

$$Z_T = \sqrt{LIMB(X)^2 + LIMB(Y)^2 + LIMB(Z)^2} \quad \text{(EQ 5.38)}$$

$$\vartheta = GEOD\_LAT \quad \text{(EQ 5.39)}$$

$$\Gamma = LOOK\_ANGLE \quad \text{(EQ 5.40)}$$

$$\omega = \text{angular rotation velocity of the Earth} \quad \text{(EQ 5.41)}$$

Then,

$$vre_{hij} = \omega Z_T \cos\vartheta \sin\Gamma \quad \text{(EQ 5.42)}$$

### 5.2.2 A2222 : Calculate velocity phases

#### FUNCTIONAL DESCRIPTION:
The projected spacecraft velocity along each line of sight and the projected Earth rotation velocity calculated in function A2221 are used to compute the velocity phase component of the total phase in a measurement.

#### INPUT:
The input parameters are:

- $\lambda_o$ - Wavelength of atmospheric line (in $\mu m$).

- $\Delta_\lambda$ - Optical path difference (OPD) for given wavelength (in $\mu m$). (one value per wavelength in CDB)

- $\left(\dfrac{\partial\Delta}{\partial\lambda}\right)$ - Derivative of the OPD for same wavelength as OPD (in $\mu m$). (one value per wavelength in CDB)

- $vps_{hij}$ - Velocity of the satellite projected along the line of sight for the given bin (in $m\ s^{-1}$). (from A2221)

- $vre_{hij}$ - Velocity of Earth's rotation projected along the line of sight for the given bin (in m s^-1). (from A2221)

### OUTPUT:
The output parameters are:

- $\Phi v_{hij}$ - Phase due to projected velocities

- $\Delta^c_\lambda$ - Corrected optical path difference.

### HYPOTHESIS OF APPLICABILITY:
The atmospheric line measured by the interferometer is Doppler shifted by the motion of the spacecraft. This shift is seen as an effective change in the optical path difference or a change in the order of interference by as much as 3000 units.

### CRITERIA OF APPLICABILITY:
Applied to all atmospheric doppler measurements.

### TRANSFORMATION EQUATION:

$$\Delta^c_\lambda = \left[ \Delta_\lambda - \lambda_0 \left( \frac{\partial \Delta}{\partial \lambda} \right) \right]$$ 

(EQ 5.43)

$$\Phi v_{hij} = \Delta^c_\lambda \times \frac{2\pi \, (vps_{hij} + vre_{hij})}{c\lambda_0} \quad \text{range} \quad [0, 2\pi]$$ 

(EQ 5.44)

### 5.2.3  A2223 : Calculate zero wind phase

#### FUNCTIONAL DESCRIPTION:
The zero wind phase is derived from the calibration phase and a relative phase difference, measured on the ground, which is read in from the CDB.

#### INPUT:
The input parameters are:

- $T$ - Time of the current measurement (for first image).

- $\Phi_{CDB_{hpq}}$ - Relative phase difference from the CDB. (one value per wavelength)

- $C_L$ - OPD neutral variation (from the CDB). This value depends on the filter or wavelength and is given by $C_L = 2\pi C^1_L \left( \frac{1}{\lambda_0} - \frac{1}{\lambda_1} \right)$.

- $\Phi 1_{CAL_{hij}}$ - Calibration phase with same binning as measurement and the most recent calibration prior to the measurement.

- $T_j$ - Time of the $\Phi 1$ calibration measurement.

- $\Phi 2_{CAL_{hij}}$ - Calibration phase with same binning as measurement and the closest calibration after the measurement.

→

- $UT_2$ - Time of the $\Phi 2$ calibration measurement.

## OUTPUT:

The output parameters are:

- $\Phi_{0_{hij}}$ - Zero wind phase in radians.

## HYPOTHESIS OF APPLICABILITY:

The relative phase difference which is stored in the CDB is $(\Phi_0^G - \Phi_1^G)$ where the 'G' stands for ground measurement and the '0' and '1' are the atmospheric line and the calibration line respectively. It is assumed that there is no optical path difference variation and that the path difference is constant between ground and space.

## CRITERIA OF APPLICABILITY:

Applied to all atmospheric doppler measurements.

## TRANSFORMATION EQUATION:

Check the measurement type and search for the frequent calibrations which bracket to the measurement. The filter groups should match giving the same window and bin sizes for the calibration and measurement. *IF the phase smoothing algorithm, is applied then the linear interpolation is between* $UT_1 = 0$, $UT_2 = 24h$

*A26,*

Get the relative phase difference from the CDB. The CDB binning must be converted to the measurement binning by averaging the CDB values in a manner similar to A2121.

Before calculating the zero wind phase check that the frequent phase calibration data matches the binning and filter number of the atmospheric measurement. *Associate the frequent phase and atmospheric measurements using the ELS lamp number and filter number as given in CDB.*

$$\text{Let } \Phi_{CDBhij} = \text{average of } \Phi_{CDB_{hpq}} \quad \forall (p,q) \text{ in } (i,j) \tag{EQ 5.45}$$

*otherwise the CDB*

*A constant, $\Delta T_{PHlimit}$) in the CDB file C_CONST sets the time interval over which to search for frequent phase measurements which bracket the atmospheric measurement.*

$$\Phi_{I_{hij}} = \Phi 1_{CAL_{hij}} + \frac{UT - UT_1}{UT_2 - UT_1} \times \left[ \Phi 2_{CAL_{hij}} - \Phi 1_{CAL_{hij}} \right] \tag{EQ 5.46}$$

$$\Phi_{0_{hij}} = \overline{\Phi_{CDBhij}} + \Phi_{I_{hij}} + C_L \tag{EQ 5.47}$$

Note: The linear interpolation of phase between calibrations may need to be changed to a second order fit to a set of calibration data. This needs to be confirmed by the instrument characterization.

### 5.2.4 A2224 : Calculate step phase and derive intrinsic phase

## FUNCTIONAL DESCRIPTION:

The phase due to the mirror stepping is added to the other phase contributions to derive the total phase without atmospheric wind phase or to derive the intrinsic phase. The velocity and zero wind phase values are added and stored for use in A31.

INPUT:

The input parameters are:

- $\Phi v_{hij}$ - Phase due to projected velocity terms.

- $\Phi_{0_{hij}}$ - Zero wind phase.

- $\delta\phi_k(\lambda)$ - Phase step for measurement of mirror motion. (from CDB, one value for each wavelength and each phase step).

- $COAE_{hpq}$ - CDB off axis effect in CDB bins (from CDB).

- $N_k$ - Number of mirror phase steps.

OUTPUT:

The output parameters are:

- $\Phi_{V0_{hij}}$ - Sum of velocity and zero wind phase. This value is catalogued for use in A31.

- $\Phi_{hijk}$ - Total or intrinsic phase for each bin.

HYPOTHESIS OF APPLICABILITY:

All the phase contributions are added together including the step due to mirror position. The off axis effect is computed for the step. (same as in A241) The phase step, $\delta\phi_k$, is nominally $\frac{\pi}{2}$ or $\frac{\pi}{4}$ for each step but varies due to errors in mirror position. Hence there is one value for each phase step. The offaxis effect is included by calculating the change in phase for each CDB bin relative to the normal incidence axis of the MI.

CRITERIA OF APPLICABILITY:

Applied to all atmospheric Doppler measurements.

TRANSFORMATION EQUATION:

$$N_{BC} = \text{number of CDB bins in measurement bin.} \qquad \text{(EQ 5.48)}$$

$$MOAE_{hij} = \frac{1}{N_{BC}}\sum_p\sum_q COAE_{hpq} \quad \forall\,(p,q)\ \text{in}\ (i,j) \qquad \text{(EQ 5.49)}$$

$$\delta\phi_{hijk} = MOAE_{hij} \times \sum_{p=1}^{k} \delta\phi_p(\lambda) \quad \text{for } k = 1, N_k \qquad \text{(EQ 5.50)}$$

$$\Phi_{v_{hij}} = MOAE_{hij} \times \Phi_{v_{hij}} \qquad \text{(EQ 5.51)}$$

*Remove.*

$$\Phi_{V0_{hij}} = \Phi v_{hij} + \Phi_{0_{hij}} \qquad \text{(EQ 5.52)}$$

$$\Phi_{hijk} = \Phi_{V0_{hij}} + \delta\phi_{hijk} \qquad \text{(EQ 5.53)}$$

## 5.3 A223 - Star removal

### 5.3.1 A2231 - Determine the angle between the observing window and the moon

FUNCTIONAL DESCRIPTION:

This algorithm flags measurement sets made too close to the moon. The closest bin to the centre of the observing window is selected and the angle between this bin and the moon is determined. If the moon is closer than $MOON_{lim}$, the measurement set is flagged and is not further processed.

*Note: There is a CDB flag which ~~allows~~ ~~it~~ selects or bypasses this algorithm.*

INPUT:

- $VECI_{hijk}$ - ECI coordinates of the bin centroids    *$k = 3$ a $5$*
- $UT_k$ - UARS standard date and time for ~~the first~~ image in the measurement.
- $N_i$, $N_j$ - Number of rows and columns in the measurement window.
- $CMOON_{lim}$ - cosine of angle $MOON_{lim}$, the minimum angle between the window and the moon for which valid data can be obtained. (from the CDB)

OUTPUT:

- $CMOON$ - cosine of $MOON$, the angle between the window and the moon. (one value per window per field of view).
- $Sflag4_h$ - Flag indicating contamination of measurement set by moonlight

HYPOTHESIS OF APPLICABILITY:

Generally applicable.

CRITERIA OF APPLICABILITY:

Applied to all sets of 4 or 8 observation images and background.

TRANSFORMATION EQUATIONS:

Select central (or near-central) image and bin:

$$\text{image:} \quad K = 3 \text{ if 4 images, 5 if 8 images per set} \tag{EQ 5.54}$$

$$\text{row:} \quad I = \frac{N_i + 1}{2} \tag{EQ 5.55}$$

$$\text{column:} \quad J = \frac{N_j + 1}{2} \tag{EQ 5.56}$$

Call orbit attitude service routine OA_EPHEM with $UT_k$, NO = 1 and MAP(1) = 2 to obtain the position of the moon. Then $xm = DATA(1,1)$, $ym = DATA(2,1)$, $zm = DATA(3,1)$ and $(x_{hIJK}, y_{hIJK}, z_{hIJK})$ the unit vector from $VECI_{hijk}$ give:

$$CMOON = x_{hIJK} \cdot xm + y_{hIJK} \cdot ym + z_{hIJK} \cdot zm \tag{EQ 5.57}$$

$$\text{If } CMOON \geq CMOON_{lim} \tag{EQ 5.58}$$

$$\text{Then } Sflag4_h = 1 \qquad \text{(EQ 5.59)}$$

$$\text{Else } Sflag4_h = 0. \qquad \text{(EQ 5.60)}$$

Note: If the image is polluted by the moon, then we process no further.

### 5.3.2 A2232: Fit curve to data points across a row of bins

#### FUNCTIONAL DESCRIPTION:

A function describing the intensity variation across the row of bins is fitted to the intensities $I_{CR_{hijk}}$ by a least squares method. The values from the fitted function corresponding to each bin are returned in $Ifit_{hijk}$. If three or more data points are missing from the row, the row is flagged with $Sflag1_{hik}$ and is not checked for stars.

#### INPUT:

The input parameters are:

- $I_{CR_{hijk}}$ - Corrected intensity for each bin and phase step. (from A2123)

- $N_j$ - The number of columns in a row.

- $Flags_{hijk}$ - Flags mapping good and bad measurement bins. (from A2114 output)

- $\Phi_{hijk}$ - Intrinsic instrument phase for zero wind, with spacecraft velocity included (from algorithm A2224)

#### OUTPUT:

The output parameters are:

- $Ifit_{hijk}$ - Intensities from the fit across a row of bins.

- $Sflag1_{hik}$ - Flag indicating which rows were not processed due to lack of data.

#### HYPOTHESIS OF APPLICABILITY:

Generally applicable.

#### CRITERIA OF APPLICABILITY:

Applied to all observation images except Filter 7.

#### TRANSFORMATION EQUATIONS:

$$Sflag1_{hik} = 1 \qquad \forall \ h, i \text{ and } k.$$

If $Flags_{hijk} = 0$ for 2 or more values of $j$ for a given value of $i$, then $Sflag1_{hik} = 0$ and the row is not checked for stars.

A function is fitted to the intensities $I_{CR_{hijk}}$ across a row of bins by a standard least squares method. The fitting routine returns a set of values of the function, $Ifit_{hijk}$, corresponding to each bin of the row.

The function used for the fit is:

**If** background or (OH full band) Filter 5a **Then**

$$Ifit_{hijk} = \text{parabolic fit with column.} \qquad \text{(account for bad bin)} \qquad \textbf{(EQ 5.61)}$$

**Else**

$$Ifit_{hijk} = a_1 + a_2 \cos \Phi_{hijk} \qquad \textbf{(EQ 5.62)}$$

where $a_1$ and $a_2$ are coefficients determined by the fitting routine.

Note: It may be necessary to account for a gradient in the phase across the field of view and to account for the 551nm filter separately. These must be confirmed in the instrument characterization.

### 5.3.3  A2233: Search for outliers from the fit

#### FUNCTIONAL DESCRIPTION:

Residuals and a standard deviation are calculated from the curve fit across a row of bins. If the standard deviation, $\sigma_{row}$, is equal to or greater than $Crt3 \times Ifit_{hijk}$, where $Ifit_{av}$ is the average value of $Ifit_{hijk}$ for the row, then the row is flagged with $Sflag1_{hik}$ and the row is not checked for stars. Any bins having a residual greater than $Crt1_j \times \sigma_{row}$ are flagged with $Sflag2_{hijk}$. Any bins having a residual greater than $Crt2 \times Ifit_{hijk}$ are flagged with $Sflag3_{hijk}$. Also, if 3 or more bins in the same row have been flagged for stars with either $Sflag2_{hijk}$ or $Sflag3_{hijk}$, then they are all flagged with $Sflag3_{hijk}$.

#### INPUT:

The input parameters are:

- $Flags_{hijk}$ - Flags mapping good and bad measurement bins. (from A2114 output)
- $N_j$ - Number of columns in a row.
- $ICR_{hijk}$ - Corrected intensity for each bin and phase step.
- $Ifit_{hijk}$ - Intensities from the fit across a row of bins.
- $Crt1_q$ - Criteria for star detection as a number of standard deviations from the fit function for each of the CDB bins in the row. (from CDB and independent of field of view)
- $Crt2$ - Criterion for rejection of data due to excessive star contamination (as a fraction of the fit value). (from CDB and independent of field of view)
- $Crt3$ - Criterion for rejection of star correction due to poor fit to the data. (from CDB and independent of field of view)
- $Sflag1_{hik}$ - Flag indicating which rows were not processed.

#### OUTPUT:

The output parameters are:

- $Res_{hijk}$ - Residuals from the fit
- $Sflag2_{hijk}$ - Flag indicating which intensities are to be corrected for the presence of stars.
- $Sflag3_{hijk}$ - Flag indicating which star-infested intensities are not correctable.
- $Flags_{hijk}$ - Flags mapping good and bad measurement bins.

## HYPOTHESIS OF APPLICABILITY:

Generally applicable. $Crt1_q$, $Crt2$ and $Crt3$ are independent of the field of view.

## CRITERIA OF APPLICABILITY:

Applied to all observation images execept for Filter 7.

## TRANSFORMATION EQUATIONS:

Combine the CDB bins to the measurement bins for the detection criteria: Average all $q$ CDB bins in each $j$ measurement bin.

$$\text{Let } \overline{Crt1}_j = \text{ average of } Crt1_q \text{ in given row.} \qquad \text{(EQ 5.63)}$$

Initialize the star flag values:

$$Sflag2_{hijk} = 1 \text{ and } Sflag3_{hijk} = 1 \quad \forall \ h, i, j \text{ and } k \qquad \text{(EQ 5.64)}$$

**If** $Sflag1_{hik} = 0$ for i **Then**

    bypass

**else**

Determine the residuals from the previous fit:

$$Res_{hijk} = \left| I_{CR_{hijk}} - Ifit_{hijk} \right| \quad \text{for } j = 1, N_j \qquad \text{(EQ 5.65)}$$

Obtain a standard deviation and the average fit value for a row:

$$\sigma_{row_{hik}} = \frac{1}{N_j} \left[ \sum_{j=1}^{N_j} (Res_{hijk})^2 \right]^{1/2} \qquad \text{(EQ 5.66)}$$

$$Ifit_{av_{hik}} = \frac{\sum_{j=1}^{N_j} Ifit_{hijk}}{N_j} \qquad \text{(EQ 5.67)}$$

Set flags:

$$\text{If } \sigma_{row_{hik}} \geq Crt3 \times Ifit_{av_{hik}} \text{ then } Sflag1_{hik} = 0. \qquad \text{(EQ 5.68)}$$

$$\text{If } \left| Res_{hijk} \right| \geq \overline{Crt1}_j \times \sigma_{row_{hik}} \text{ then } Sflag2_{hijk} = 0. \qquad \textbf{(EQ 5.69)}$$

$$\text{If } \left| Res_{hijk} \right| \geq Crt2 \times Ifit_{hijk} \text{ then } Sflag3_{hijk} = 0. \qquad \textbf{(EQ 5.70)}$$

Reject rows which cannot be processed further:

$$\text{If } Sflag1_{hik} = 0 \text{ then } Sflag3_{hijk} = 0 \quad \forall \, j \text{ in row } i \qquad \textbf{(EQ 5.71)}$$

Reject bins which cannot be processed further:

$$\text{If } \sum_{j=1}^{N_j} (Sflag2_{hijk} \times Sflag3_{hijk}) \leq (N_j - 2) \text{ then } Sflag3_{hijk} = 0 \qquad \textbf{(EQ 5.72)}$$

The bin flags are updated for bad bins due to stars:

$$Flags_{hijk} = Flags_{hijk} \times Sflag3_{hijk} \text{ for each bin.} \qquad \textbf{(EQ 5.73)}$$

### 5.3.4  A2234: Replace data point if affected by star

#### FUNCTIONAL DESCRIPTION:

The data points affected by stars, as indicated by $Sflag2_{hijk}$, are removed and a curve is fit to the remaining data points using the same functions as in A2232. The removed data points are replaced by their corresponding values in the fitted curve.

#### INPUT:

The input parameters are:

- $Sflag2_{hijk}$ - Flag indicating which intensities are to be corrected for the presence of stars (from A2233)
- $Sflag3_{hijk}$ - Flag indicating which bins cannot be corrected.
- $ICR_{hijk}$ - Corrected intensity for each bin and phase step.
- $Res_{hijk}$ - Residuals from the fit of algorithm A2233.
- $N_j$ - Number of columns in a row.
- $\sigma^2_{CR_{hijk}}$ - Initial error estimate of signal level from A2123.

#### OUTPUT:

The output parameters are:

- $I_{S_{hijk}}$ - Corrected intensity for each bin and phase step (now also corrected for star contamination).
- $\sigma^2_{S_{hijk}}$ - Error estimate of corrected intensity.

HYPOTHESIS OF APPLICABILITY:

Generally applicable.

CRITERIA OF APPLICABILITY:

Applied to all observation images except for Filter 7.

TRANSFORMATION EQUATIONS:

If $Sflag3_{hijk} = 0$ then make no correction for that bin.

If $Sflag2_{hijk} = 0$ then for the affected bin only

$$I_{S_{hijk}} = I_{CR_{hijk}} - Res_{hijk} \qquad \textbf{(EQ 5.74)}$$

$$\sigma^2_{S_{hijk}} = \sigma^2_{CR_{hijk}} \left( 1 + \left| \frac{Res_{hijk}}{I_{CR_{hijk}}} \right| \right) \qquad \textbf{(EQ 5.75)}$$

Note: Only the bins affected by stars are corrected. The remaining bins which need no correction are saved as:

$$I_{S_{hijk}} = I_{CR_{hijk}} \qquad \textbf{(EQ 5.76)}$$

$$\sigma^2_{S_{hijk}} = \sigma^2_{CR_{hijk}} \qquad \textbf{(EQ 5.77)}$$

## 5.4 A224 - Roll effect attitude correction

### 5.4.1 A2241: Determine whether correction is needed

FUNCTIONAL DESCRIPTION:

When the spacecraft experiences a roll or similar movement the line of view through the atmosphere is displaced changing the value of the observed radiance. This phenomenon generates significant errors if it occurs during the multiple image measurement sequence. Narrow emission features are most sensitive to this effect. Corrections are only made for mesospheric $O^1S$ and OH (night time measurements) and for large $\Delta Z$.

INPUT:

The input parameters are:

- $N_h$ - Number of fields of view (=2).
- $N_i$ - Number of rows in measurement window.
- $N_j$ - Number of columns in measurement window.
- $N_k$ - Number of phase steps in measurement.
- $Z_{hijk}$ - Altitude at the tangent point for each bin.
- $ZR_{hi}$ - Reference altitude profile from A2213.

- $\Delta Z_{max}$ - Maximum change in altitude allowed before the roll correction is done. (from CDB)

**OUTPUT:**

The output parameters are:

- $\Delta Z_{hijk}$ - Altitude variation of bins relative to image one.

**HYPOTHESIS OF APPLICABILITY:**

The spacecraft movement is not a continuous roll. It contains small random movements which occur during the exposure time. The frequency of these movements is larger than the sampling rate of the attitude. The value of $\Delta Z$ is an average or mean change during the exposure time. Consequently it is assumed that the amplitude of the random movements is smaller than the global mean change of attitude.

**CRITERIA OF APPLICABILITY:**

This routine is always called *for measurements with more than one image.*

**TRANSFORMATION EQUATION:**

*compute for all filters and for both day and night.*

If the measurement is either nighttime observations of $O(^1S)$ or OH

then compute the altitude variation as follows:

$$\Delta Z_{hijk} = 0 \quad \forall \ h, i, j \text{ and } k = 1 \qquad \text{(EQ 5.78)}$$

$$\Delta Z_{hijk} = Z_{hijk} - Z_{hij1} \quad \forall \ h, i, j \text{ and } k = 2, N_k \qquad \text{(EQ 5.79)}$$

$$\text{If any } |\Delta Z_{hijk}| \geq \Delta Z_{max} \text{ execute A2242-3.} \qquad \text{(EQ 5.80)}$$

otherwise bypass the roll correction and do NOT execute A2242-3.

### 5.4.2 A2242: Calculate partial derivative of bin intensity

**FUNCTIONAL DESCRIPTION:**

Compute the partial derivative with respect to altitude by finding the two partial derivatives with respect to row. The final derivative is in altitude on each vertical column.

**INPUT:**

The input parameters are:

- $\text{Flags}_{hijk}$
- $N_h$ - Number of fields of view (=2).
- $N_i$ - Number of rows in measurement window.
- $N_j$ - Number of columns in measurement window.
- $N_k$ - Number of phase steps in measurement.

- $IS_{hijk}$ - Measured signal for each bin (in rayleigh) corrected for dark current and star contamination.

- $Z_{hijk}$ - Tangent altitude of each bin.

## OUTPUT:
The output parameters are:

- $\left(\dfrac{\partial I_s}{\partial Z}\right)_{hijk}$ - Partial derivative of the measured signal with respect to altitude for each vertical column.

## HYPOTHESIS OF APPLICABILITY:
The spacecraft movement is not a continuous roll. It contains small random movements which occur during the exposure time. The frequency of these movements is larger than the sampling rate of the attitude. The value of $\Delta Z$ is an average or mean change during the exposure time. Consequently it is assumed that the amplitude of the random movements is smaller than the global mean change of attitude.

## CRITERIA OF APPLICABILITY:
This routine is always called if the roll attitude correction is to be done.

## TRANSFORMATION EQUATION:

$$\left(\frac{\partial I_s}{\partial Z}\right)_{hijk} = \left(\frac{\partial I_S^{hijk}}{\partial i}\right) \bigg/ \left(\frac{\partial Z_{hijk}}{\partial i}\right) \quad \forall \ h, i, j \text{ and } k \qquad \text{(EQ 5.81)}$$

**If** no correction due to bad bin **Then**

$$\text{set } \frac{\partial I}{\partial Z} = 0$$

In order to compute the above derivatives use a smoothing function such as a second order fit to the 7 data points (3 on each side) surrounding the current point. The corrections applied should be small and so the derivatives must be smooth inorder not to introduce excessive error. If there is a bad bin, then use only six data points.

### 5.4.3  A2243: Correct the data for roll effect

## FUNCTIONAL DESCRIPTION:
The measured signal is corrected for the roll effect by adjusting the value according to the partial derivatives and the variations in altitude.

## INPUT:
The input parameters are:

- $N_h$ - Number of fields of view (=2).

- $N_i$ - Number of rows in measurement window.

- $N_j$ - Number of columns in measurement window.

- $N_k$ - Number of phase steps in measurement.

- $\Delta Z_{hijk}$ - Variation in altitude relative to first image.

- $IS_{hijk}$ - Measured signal in each bin (in rayleighs).

- $\left(\dfrac{\partial I_S}{\partial Z}\right)_{hijk}$ - Partial derivative of measured signal in each column.

- $\sigma^2_{S_{hijk}}$ - Error estimate of intensity from A2234.

## OUTPUT:

The output parameters are:

- $IR_{hijk}$ - Corrected data referenced to image one.

- $\sigma^2_{R_{hijk}}$ - Error estimate of intensity.

## HYPOTHESIS OF APPLICABILITY:

The spacecraft movement is not a continuous roll. It contains small random movements which occur during the exposure time. The frequency of these movements is larger than the sampling rate of the attitude. The value of $\Delta Z$ is an average or mean change during the exposure time. Consequently it is assumed that the amplitude of the random movements is smaller than the global mean change of attitude.

## CRITERIA OF APPLICABILITY:

This routine is always called if the roll attitude correction is to be done.

## TRANSFORMATION EQUATION:

$$I_{R_{hijk}} = I_{S_{hijk}} - \left(\frac{\partial I_s}{\partial Z}\right)_{hijk} \Delta Z_{hijk} \quad \forall \; h, i, j \text{ and } k. \tag{EQ 5.82}$$

$$\sigma^2_{R_{hijk}} = \sigma^2_{S_{hijk}} \left( 1 + \frac{\left| \left(\frac{\partial I_s}{\partial Z}\right)_{hijk} \Delta Z_{hijk} \right|}{\left| I_{S_{hijk}} \right|} \right) \tag{EQ 5.83}$$

The first image is not corrected as all $\Delta Z_{hijk}$ are zero for $k=1$ which is the reference image.

# CHAPTER 6

# A23 - Background subtraction

*Note: The LICVB file is the dark current-subtracted image converted to Rayleigh but with NO wavelength correction.*

## 6.1 A231 - Wavelength conversion

### 6.1.1 A2311: WAVELENGTH CONVERSION 557.7 nm DAY

#### FUNCTIONAL DESCRIPTION:

This algorithm receives as its data input the dark current and star corrected background measurement at 551 nm. It fits to these data a two-component model consisting of atmospheric Rayleigh scattering and baffle scattering. The Rayleigh scattering component is modelled with an exponential function (see below), having two adjustable parameters; the Rayleigh scattering at 85 km $S(Z_o)$ and its scale height H. The baffle scattering is modelled by using an empirical model from the CDB, and adjusting it by a single scaling factor K. Thus the application of the two-component model yields four parameters, $S(Z_o)$, H, K1 and K2, which represent the background signal at 551 nm. This must now be converted to the background signal at 557 nm, which in the general case is done by applying separate wavelength conversion factors for Rayleigh, S(557)/S(551), and baffle, B(557)/B(551), scattering (called below the Model method). A simpler direct method is to be optionally available; this involves using an average wavelength conversion factor WCAV for both components. For this direct approach the background is not separated into its two components --- the measured background is simply scaled with this average wavelength conversion factor WCAV.

#### INPUT:

The input parameters are:

- *BMsw* - background model switch (run time parameter).

- $O1B_{hij}$ - dark current and starlight corrected background image at 551 nm, in rayleighs.

- $Z_{hij1}$ - altitude profile associated with the background image. (k=1 since the background measurements have only 1 image)
- $Z_s$ - altitude of the spacecraft.
- $B^m{}_{hij}(Z_s)$ - baffle scattering model (see definition below). (Model parameters stored in the CDB)
- $Z_0$ - reference altitude for the Rayleigh Scattering profile. (from the CDB)
- $Z_{ref}$ - reference altitude (from CDB) which defines baffle scattering threshold region.
- $\dfrac{S(557)}{S(551)}$ - rayleigh scattering ratio obtained from the CDB.
- $\dfrac{B(557)}{B(551)}$ - baffle scattering ratio obtained from the CDB.
- WCAV - average wavelength conversion factor obtained from the CDB.
- $a_{shq}$, $b_{shq}$, $c_{shq}$ - coefficients of the baffle scattering model. (from the CDB)

## OUTPUT:

The output parameters are:

- $O1B557_{hij}$ - background image converted to 557.7 nm.
- $S(Z_0)$ - Rayleigh scattering at Z0.
- $H$ - scale height of apparent Rayleigh scattering.
- $K_1$, $K_2$ - baffle scattering normalization parameters.

## HYPOTHESIS OF APPLICABILITY:

The Model method assumes that the baffle scattering model described can be determined in orbit, and that baffle scattering can be separated from Rayleigh scattering.

## CRITERIA OF APPLICABILITY:

By DAY we mean that the daytime aperture stop is in position. Experience may show that the single scaling factor WCAV (the direct method) will suffice. However the full modelled analysis is preferred if it is found that meaningful values of Rayleigh scattering can be obtained.

## TRANSFORMATION EQUATIONS:

The Rayleigh Scattering component is described by:

$$S_{hij} = S(Z_0)\, e^{-\dfrac{(Z_{hij} - Z_0)}{H}} \qquad \text{(EQ 6.1)}$$

The Baffle Scattering component is described by ( Value from CDB):

$$Z_{OB} = 300 \text{ km} \qquad \text{(EQ 6.2)}$$

$$B_{hij} = K_{1_h} B^m_{hij}(Z_s) + K_{2_h} \qquad \text{(EQ 6.3)}$$

$$\text{where } B^m_{hij}(Z_s) = a_{shj} + b_{shj}(Z_{OB} - Z_{hij}) + c_{shj}(Z_{OB} - Z_{hij})^2 \qquad \text{(EQ 6.4)}$$

Each set of coefficients (ashq,bshq,cshq) is obtained from the CDB and combined into the measurement bins for all $q$ in $j$ as in function A2113 to give the $(a_{shj}, b_{shj}, c_{shj})$.

The coefficients are defined at each of 6 altitudes for the space craft. The coefficients are linearly interpolated in spacecraft altitude to the current altitude $Z_s$.

The following table gives the format of the coeffcients. This gives 40 columns by 2 FOVs by 6 altitudes or 480 triplets.

| $Z_s$ | FOV1 | FOV2 |
|-------|------|------|
| 575 | $(a111,b111,c111), (a112,b112,c112)...$ | $(a121,b121,c121), (a122,b122,c122)...$ |
| 585 | $(a211,b211,c211), (a212,b212,c212)...$ | $(a221,b221,c221), (a222,b222,c222)...$ |
| 595 | $(a311,b311,c311), (a312,b312,c312)...$ | $(a321,b321,c321), (a322,b322,c322)...$ |
| 605 | $(a411,b411,c411), (a412,b412,c412)...$ | $(a421,b421,c421), (a422,b422,c422)...$ |
| 615 | $(a511,b511,c511), (a512,b512,c512)...$ | $(a521,b521,c521), (a522,b522,c522)...$ |
| 625 | $(a611,b611,c611), (a612,b612,c612)...$ | $(a621,b621,c621), (a622,b622,c622)...$ |

Two wavelength conversion processes are switch selectable:

**IF** BMsw = 0

**Then** Direct:

$$O1B557_{hij} = O1B_{hij} \times WCAV \qquad \text{(EQ 6.5)}$$

**Else** Modelled: (If $BMsw = 1$)

Note: If this fails, then use direct.

$$\text{Interpolate} \qquad (a_{shj}, b_{shj}, c_{shj}) \qquad \forall \ h \text{ and } j \text{ at } Z_s \qquad \text{(EQ 6.6)}$$

Fit    (linear least mean squares)

$$Z_{ref} = 130 \text{ km} \qquad \text{(EQ 6.7)}$$

$$O1B_{hij} = K_{1_h} B^m_{hij}(Z_s) + K_{2_h} \quad \text{for all } Z_{hij} > Z_{ref} \qquad \text{(check for bad bins)} \qquad \text{(EQ 6.8)}$$

$$\text{Calculate} \qquad B_{hij} \text{ from } K_{1_h} \text{ and } K_{2_h} \text{ for all bins.} \qquad \text{(EQ 6.9)}$$

$$O1B^b_{hij} = O1B_{hij} - B_{hij} \qquad \text{(EQ 6.10)}$$

Fit     (linear least mean squares)

$$\ln (O1B_{hij}^b) = \ln (S(Z_0)) - \frac{1}{H}(Z_{hij} - Z_0) \qquad \text{(check for bad bins)} \qquad \textbf{(EQ 6.11)}$$

Calaculate     $S_{hij}$ using $S(Z_0)$ and $H$     **(EQ 6.12)**

$$O1B557_{hij} = S_{hij} \times \frac{S(557)}{S(551)} + B_{hij} \times \frac{B(557)}{B(551)} \qquad \textbf{(EQ 6.13)}$$

Note: This function will be refined after launch once atmospheric data have been obtained. Part of the initial operations plan will be dedicated to verifying the background corrections.

### 6.1.2   A2312: WAVELENGTH CONVERSION 630.0 nm DAY

**FUNCTIONAL DESCRIPTION:**
This algorithm receives as its data input the dark current and star corrected background image at 551 nm. This is converted to a background image at 630.0 nm.

**INPUT:**
The input parameters are:

- $O1B_{hij}$ - dark current and star corrected background image at 551 nm, in Rayleigh.

- $\frac{B(630)}{B(551)}$ - baffle scattering ratio obtained from the CDB.

**OUTPUT:**
The output parameters are:

- $O1B630_{hij}$ - background image converted to 630.0 nm, in rayleighs.

**HYPOTHESIS OF APPLICABILITY:**
This algorithm rests on the assumption that the ratio between the baffle scattering at 551 nm and that at 630 nm is a constant.

**CRITERIA OF APPLICABILITY:**
By DAY we mean that the daytime aperture stop is in position. Since the 630 nm emission is only at high altitude (unlike the 557.7 nm emission) there is no attempt to recover the Rayleigh scattered light. Hence the direct method is used.

**TRANSFORMATION EQUATION:**

$$O1B630_{hij} = 01B_{hij} \times \frac{B(630)}{B(551)} \qquad \textbf{(EQ 6.14)}$$

### 6.1.3  A2313: WAVELENGTH CONVERSION 732.0 nm O⁺ DAY

#### FUNCTIONAL DESCRIPTION:

This algorithm receives as its data input the background measurement obtained from the OH Doppler channel. This is converted to a background image at 732 nm.

#### INPUT:

The input parameters are:

- $OHD_{hij}$ - background image at 737 nm (which is the OH Doppler channel), in rayleighs.

- $\dfrac{B(732)}{B(737)}$ - baffle scattering ratio obtained from the CDB.

#### OUTPUT:

The output parameters are:

- $OHD732_{hij}$ - background image converted to 732 nm, in rayleighs.

#### HYPOTHESIS OF APPLICABILITY:

Generally applicable.

#### CRITERIA OF APPLICABILITY:

By DAY we mean that the daytime aperture is in position.

#### TRANSFORMATION EQUATION:

$$OHD732_{hij} = OHD_{hij} \times \frac{B(732)}{B(737)}$$  (EQ 6.15)

### 6.1.4  A2314: WAVELENGTH CONVERSION 557.7 nm NIGHT MESOSPHERE

#### FUNCTIONAL DESCRIPTION:

This algorithm receives as its data input the 551 nm background measurement, from which the dark current and discrete stars have been subtracted. From this image the diffuse starlight background is subtracted, which is done using the average signal level across the top row of bins, which is just above the $NO_2$ continuum layer, and is denoted ES551. The image with the starlight subtracted is saved as an image of the $NO_2$ continuum. Where a 557 nm thermospheric measurement follows the mesospheric one, the same starlight level is used for both.

#### INPUT:

- $O1B_{hij}$ - dark current and star corrected background image at 551 nm, in rayleighs.

- $\dfrac{NO_2(557)}{NO_2(551)}$ - wavelength conversion factor for NO2, obtained from the CDB.

- $\dfrac{ST(557)}{ST(551)}$ - wavelength conversion factor for starlight, obtained from CDB.

**OUTPUT:**

- $O1B557_{hij}$ - background image converted to 557 nm, in rayleighs.

- $NO2_{hi}$ - $NO_2$ continuum image, to be stored as a data product, averaged to one column.

**HYPOTHESIS OF APPLICABILITY:**

This method assumes that an averaged value of diffuse starlight background, obtained just above the $NO_2$ layer is adequate for the subtraction of starlight from all other bins. It also assumes that the background signal, with starlight subtracted, is a good representation of the $NO_2$ continuum. There will be a variation in the signal due to diffuse star light from the highest altitude line of sight to the lowest altitude line of sight due to varying absorption. It is not known how large an effect this will be and it may be necessary to account for this after atmospheric data is available for analysis.

**CRITERIA OF APPLICABILITY:**

By NIGHT we mean that the nighttime aperture is in use.

**TRANSFORMATION EQUATIONS:**

The top row or highest altitude row of the image is row i=1. Hence the estimate of the diffuse star background is taken as the average across row 1 for both fields of view.

$$N_{gb} = \sum_j Flags_{hij} \tag{EQ 6.16}$$

$$ES551_h = \frac{1}{N_j} \sum_j O1B_{hij} \tag{EQ 6.17}$$

**If** $N_{gb} = 0$ then

$$ES551_h = 0 \tag{EQ 6.18}$$

set error message.

$$NO2_{hi} = \frac{1}{N_j} \sum_j [O1B_{hij} - ES551_h] \tag{EQ 6.19}$$

$$O1B557_{hij} = [O1B_{hij} - ES551] \times \frac{NO_2(557)}{NO_2(551)} + ES551 \times \frac{ST(557)}{ST(551)} \tag{EQ 6.20}$$

### 6.1.5 A2315: WAVELENGTH CONVERSION 630 nm NIGHT

**FUNCTIONAL DESCRIPTION:**

At night, the background for the thermospheric emission will be the diffuse starlight. If the 630 nm measurement is made by itself, a background image at 551 nm will be acquired with it. If the 630 nm measurement is made in conjunction with a 557 nm measurement, then the same starlight will be used for both.

<u>INPUT:</u>

The input parameters are:

- $O1B_{hij}$ - dark current and star corrected background image at 551 nm.

- $\dfrac{ST(630)}{ST(551)}$ - wavelength conversion factor for starlight obtained from the CDB.

<u>OUTPUT:</u>

The output parameters are:

- $O1B630_{hij}$ - background image converted to 630 nm.

<u>HYPOTHESIS OF APPLICABILITY:</u>

It is assumed that starlight is the only background signal for the 630 nm emission at night.

<u>CRITERIA OF APPLICABILITY:</u>

By NIGHT we mean that the nighttime aperture is in use.

<u>TRANSFORMATION EQUATIONS:</u>

$$O1B630_{hij} = 01B_{hij} \times \frac{B(630)}{B(551)} \qquad \textbf{(EQ 6.21)}$$

### 6.1.6  A2316: WAVELENGTH CONVERSION OH NIGHT

<u>FUNCTIONAL DESCRIPTION:</u>

This algorithm receives as its data input a dark current and discrete star corrected background measurement at 735 nm. First of all the diffuse starlight background is subtracted from this image, using the average of the row of bins just above the $NO_2$ and OH emission layers in each of the windows. By using a wavelength conversion factor and noting that the starlight level in the 735 nm window is twice that in the other windows because of the wedge, the corrected image is converted to background images at 734 and 737 nm. The background image with starlight subtracted is saved as an image of the $NO_2$ continuum.

<u>INPUT:</u>

The input parameters are:

- $OHB_{hij}$ - the average of the eight dark current and star corrected background images at 735 nm --- rayleighs.

- $\dfrac{NO_2(737)}{NO_2(735)}, \dfrac{NO_2(734)}{NO_2(735)}$ - wavelength conversion factors for NO2 obtained from the CDB.

- $\dfrac{ST(734)}{ST(735)}, \dfrac{ST(737)}{ST(735)}$ - wavelength conversion factors for starlight obtained from the CDB.

### OUTPUT:

The output parameters are:

- $OHB734_{hij}$ - background image converted to 734 nm --- rayleighs.

- $OHB737_{hij}$ - background image converted to 737 nm --- rayleighs.

- $NO2OH_{hi}$ - $NO_2$ continuum image to be stored as a data product, averaged to a single column.

### HYPOTHESIS OF APPLICABILITY:

It is assumed that diffuse starlight and the $NO_2$ continuum are the only background sources for OH at night.

### CRITERIA OF APPLICABILITY:

By NIGHT we mean that the nighttime aperture is in use.

### TRANSFORMATION EQUATIONS:

$$ES735_h = \frac{1}{N_j}\sum_j OHB_{h1j} \quad \text{(check } Flags_{hij}\text{)} \qquad \text{(EQ 6.22)}$$

*with good data*

where $OHB_{h1j}$ is the measurement at the highest altitude row in background image.

$$NO2_{hij} = OHB_{hij} - ES735 \qquad \text{(EQ 6.23)}$$

$$OHB734_{hij} = NO2_{hij}\frac{NO_2(734)}{NO_2(735)} + \frac{ES735}{2}\frac{ST(734)}{ST(735)} \qquad \text{(EQ 6.24)}$$

$$OHB737_{hij} = NO2_{hij}\frac{NO_2(737)}{NO_2(735)} + \frac{ES735}{2}\frac{ST(737)}{ST(735)} \qquad \text{(EQ 6.25)}$$

$$NO2OH_{hi} = \frac{1}{N_j}\sum_j NO2_{hij} \qquad \text{(EQ 6.26)}$$

Note: For the above, one must check for bad bins.

## 6.2 A232: Interpolate to measurement reference altitude profile

### FUNCTIONAL DESCRIPTION:

The background image is taken at a time and location different from the associated measurement. Consequently the altitude profile of the background will not be sampled at the same altitudes as the measurement. This function interpolates the background profile using a NAG least squares spline routine to the measurement altitudes.

### INPUT:

The input parameters are:

- $B_{gnd_{hij}}$ - The wavelength corrected background from A231.
- $ZB_{hij}$ - The altitude profile of the background image.
- $Z_{hijk}$ - The altitude profile of the current measurement.

### OUTPUT:

The output parameters are:

- $BgndZ_{hijk}$ - The background interpolated to the measurement altitudes.

### HYPOTHESIS OF APPLICABILITY:

It is assumed that the background image taken prior to the measurement can be interpolated to the measurment location.

### CRITERIA OF APPLICABILITY:

None.

### TRANSFORMATION EQUATION:

The vertical profile in each column and in each field of view is fit using a NAG cubic spline least squares routine (E02BAF) as a function of altitude. That is $ZB_{hi}$ and $Bgnd_{hi}$ vectors are input for each $j$.

The new values of background are then calculated for each $Zh_{ijk}$ to give $BgndZ_{hijk}$.

Note: As an alternative to the simple cubic spline interpolation, a bicubic spline could be used to fit the entire background image. This would introduce some dependence from one column to the next and the effects of this would need to be studied.

## 6.3 A233: Background Subtraction

### FUNCTIONAL DESCRIPTION:

The wavelength and altitude corrected background is subtracted bin by bin from the measured data.

### INPUT:

The input parameters are:

- $I_{R_{hijk}}$ - Corrected image intensities from A2243.

- $\sigma^2_{R_{hijk}}$ - Error estimate for corrected data from A2243.

- $BgndZ_{hijk}$ - Background at reference altitudes from A232.

### OUTPUT:
The output parameters are:

- $I_{B_{hijk}}$ - Background corrected intensity images.

- $\sigma^2_{B_{hijk}}$ - Error estimate of background corrected data.

### HYPOTHESIS OF APPLICABILITY:
Assumes all corrections have been made to the data and that the appropriate background has been selected and corrected for the given measurement.

### CRITERIA OF APPLICABILITY:
None.

### TRANSFORMATION EQUATION:

$$I_{B_{hijk}} = I_{R_{hijk}} - BgndZ_{hijk}$$ 

(EQ 6.27)

$$\sigma^2_{B_{hijk}} = \sigma^2_{R_{hijk}} \left( 1 + \left| \frac{BgndZ_{hijk}}{I_{R_{hijk}}} \right| \right)$$ 

(EQ 6.28)

## 6.4  A234: Filter transmission correction

### FUNCTIONAL DESCRIPTION:
The signal level in Rayleigh is corrected for the fact that the emission line is not located at the peak of the filter. The correction is made using the filter transmittance (normalized to unity at the filter peak) at the location of the emission line.

### INPUT:
The input parameters are:

- $I_{B_{hijk}}$ - Corrected signal level (in Rayleigh) from function A233.

- $MT_{hij}$ - Filter transmittances for the measurement bins, from A2113.

- $\sigma^2_{B_{hijk}}$ - Error estimate of background corrected data.

### OUTPUT:
The output parameters are:

- $I_{hijk}$ - emission line signal level (in Rayleighs), corrected for filter transmission.

- $\sigma^2_{I_{hijk}}$ - Error estimate of emission line signal.

## HYPOTHESIS OF APPLICABILITY:

This process assumes that all dark current and background have been subtracted, so that the signal arises purely from an isolated emission line.

## CRITERIA OF APPLICABILITY:

This process is applied to all observation images except $O_2$.

## TRANSFORMATION EQUATION:

$$I_{hijk} = \frac{I_{B_{hijk}}}{MT_{hij}}$$

(EQ 6.29)

$$\sigma^2_{I_{hijk}} = \frac{\sigma^2_{B_{hijk}}}{(MT_{hij})^2}$$

(EQ 6.30)

Note: The transmittance of the filter may change with filter temperature. The filter transmission correction may need to be changed after the insturment characterization to account for this effect.

# CHAPTER 7

# A24 - Extract calibration parameters

Note: The phase, visibility and dark current obtained in the frequent calibrations is to be saved for the use by other algorithms. Phase and visibility are computed in this funciton. Dark current images do not need to be processed further but must be saved.

## 7.1 A241: Calculate step phase

### FUNCTIONAL DESCRIPTION:
The phase due to the mirror stepping is computed for the calibration measurement.

### INPUT:
The input parameters are:

- $\delta\phi_k(\lambda)$ - Phase step of mirror motion (from CDB, one value for each wavelength and each phase step, approximately or ).
- $COAE_{hpq}$ - CDB Off Axis Effect in CDB bins (from CDB). These are multiplicative values relative to the normal incidence value at the MI optical axis.
- $N_k$ - Number of mirror phase steps.

### OUTPUT:
The output parameters are:

- $\delta\Phi_{CAL_{hijk}}$ - Calibration phase step.

### HYPOTHESIS OF APPLICABILITY:
The off axis effect is determined during the calibration of the instrument and is set up as an array of values relative to the optical centre of the Michelson. Thus the effective step

size is changed by this relative amount to account for the longer path length for off axis rays.

### CRITERIA OF APPLICABILITY:
None.

### TRANSFORMATION EQUATION:
The measurement binning of the off axis effect map is calculated in the same manner as for the CDB bin conversion in A2113.

$$N_{BC} = \text{number of CDB bins in measurement bin.} \qquad \text{(EQ 7.1)}$$

$$MOAE_{hij} = \frac{1}{N_{BC}} \sum_{q}^{p} COAE_{hpq} \quad \forall\, (p, q) \text{ in } (i, j) \qquad \text{(EQ 7.2)}$$

$$\delta\Phi_{CAL_{hijk}} = MOAE_{hij} \times \sum_{p=1}^{k} \delta\phi_p(\lambda) \quad \text{for } k = 1, N_k \qquad \text{(EQ 7.3)}$$

assumes phase steps are cummulative.

## 7.2 A242 : Calculate calibration vector

### FUNCTIONAL DESCRIPTION:
The data from the frequent phase calibration is analyzed using either the Multiple Linear Regression tool (A3131) or the Multiple Point Algorithm (A3132) to derive J1, J2 and J3 values.

### INPUT:
The input parameters are:

- $I_{CAL_{hijk}}$ - Phase calibration image which is dark current corrected.

- $\sigma^2_{CAL_{hijk}}$ - Error estimate of phase calibration data.

- $\delta\Phi_{CAL_{hijk}}$ - Calibration phase step.

- $N_i$ - Number of rows in image.

- $N_j$ - Number of columns in image.

- $N_k$ - Number of mirror phase steps.

- *mode* - The mode of weighting used in the MLR or MPA fitting. If *mode*=0 then equal weighting, if *mode*=1 then statistical weighting. (This could be a run time parameter.)   *In CDB C-Const*

### OUTPUT:
The output parameters are:

- $J1_{hij}$, $J2_{hij}$ and $J3_{hij}$ - Fit values for each bin.

- $\sigma^2_{J1_{hij}}$, $\sigma^2_{J2_{hij}}$ $\sigma^2_{J3_{hij}}$ - Error estimate of J values.

## HYPOTHESIS OF APPLICABILITY:

The calibration intensities must be sent to the MLR or MPA routines bin by bin since these routines operate on vectors of data representing a single phase measurement or a stacked set of phase measurements. Each J1, J2 and J3 for each bin must be used to compute the phase and visibility in A243.

## CRITERIA OF APPLICABILITY:

None.

## TRANSFORMATION EQUATION:

Set $N_p = N_k$, order=0, $N_{term}$=2 and $jterm_q = q$ for $q$=1,$N_{term}$. (These are all local variables used in the call to A31313).

$$
\text{Set} \left\{
\begin{array}{c}
u^f_p = 1 \\[6pt]
\phi^f_p = \delta\Phi_{CAL_{hijp}} \\[6pt]
t^f_p = \phi^f_p \\[6pt]
B^f_p = I_{CAL_{hijp}} \\[6pt]
\text{If } mode = 0 \text{ then } \sigma_p = 1 \\[6pt]
\text{If } mode = 1 \text{ then } \sigma_p = \sqrt{\sigma^2_{CAL_{hijp}}}
\end{array}
\right\}
\quad \text{for } p = 1, N_p \quad \text{(check for bad bins)} \quad \textbf{(EQ 7.4)}
$$

**If** bad bin, then

    set J1, J2, J3 = 1

    $\sigma = 1$

    do not call MLR

**Else**

Call A31313 to compute MLR fit. Then,

$$
J1_{hij} = b_0 \qquad \sigma^2_{J1_{hij}} = \sigma^2_{b_0} \qquad\qquad \textbf{(EQ 7.5)}
$$

$$
J2_{hij} = b_1 \qquad \sigma^2_{J2_{hij}} = \sigma^2_{b_1} \qquad\qquad \textbf{(EQ 7.6)}
$$

$$
J3_{hij} = b_2 \qquad \sigma^2_{J3_{hij}} = \sigma^2_{b_2} \quad \forall\ h, i, \text{ and } j. \qquad \textbf{(EQ 7.7)}
$$

Alternatively use $B^f_p$ as the input to the MPA routine and compute J1, J2 and J3.

Note: Both the MPA and MLR routines are included here as possible functions to use. It is intended that the function which executes the fastest and provides the most accurate results will be used. It is not clear at this time which one will be best.

## 7.3  A243 : Extract phase and visibility

### FUNCTIONAL DESCRIPTION:

The values of J1, J2 and J3 for each bin in the calibration image are used to derive calibration phase and visibility. The calibration values obtained here must be stored for later use by the measurement exctraction routines.

### INPUT:

The input parameters are:

- $J1_{hij}$, $J2_{hij}$, $J3_{hij}$ - J values from A242.

- $\sigma^2_{J1_{hij}}$, $\sigma^2_{J2_{hij}}$  $\sigma^2_{J3_{hij}}$ - Error estimate of J values from A242.

### OUTPUT:

The output parameters are:

- $\Phi_{CAL_{hij}}$ - Frequent calibration phase.

- $\sigma^2_{\Phi CAL_{hij}}$ - Error estimate for phase.

- $V_{CAL_{hij}}$ - Visibility from frequent calibration.

- $\sigma^2_{VCAL_{hij}}$ - Error estimate for visibility.

- $\sigma_Q$ - std dev.

### HYPOTHESIS OF APPLICABILITY:

The frequent phase values are used in the processing of the flight data, however, the visibility values are simply stored and are used off line to look at the instrument visibility in conjunction with the infrequent calibration data.

### CRITERIA OF APPLICABILITY:

None.

### TRANSFORMATION EQUATION:

$$V_{CAL_{hij}} = \frac{\sqrt{J2^2_{hij} + J3^2_{hij}}}{J1_{hij}}$$

(EQ 7.8)

$$\sigma_{VCAL_{hij}} = \frac{1}{J1_{hij}} \left[ \frac{\sigma^2_{J2_{hij}} J2^2_{hij} + \sigma^2_{J3_{hij}} J3^2_{hij}}{J2^2_{hij} + J3^2_{hij}} + \sigma^2_{J1_{hij}} V^2_{CAL_{hij}} \right]^{1/2}$$

(EQ 7.9)

range of visibility values is [0,1].

$$\Phi_{CAL_{hij}} = \tan^{-1}\left(\frac{J3_{hij}}{J2_{hij}}\right) \qquad \forall \ h, i \text{ and } j \qquad \textbf{(EQ 7.10)}$$

$$\sigma_{\Phi CAL_{hij}} = \frac{\left[\sigma_{J2_{hij}}^2 J3_{hij}^2 + \sigma_{J3_{hij}}^2 J2_{hij}^2\right]^{1/2}}{J2_{hij}^2 + J3_{hij}^2} \qquad \textbf{(EQ 7.11)}$$

$\rightarrow$ *check notation on range*

range of phase values is $[0,2\pi]$  *O included in range ($2\pi$ not)*

Note: The proper form of the inverse tangent function must be used in order to ensure the proper range of phase values is returned. That is the function must use both sign and magnitude of J3 and J2 inorder to determine the angle correctly.

Note: The catalogued output data interface document specifies a quality parameter for each column of output. Since the phase is the value used by the algorithms the quality flag is based on the phase error as:

$$\sigma_{Q_{hij}} = \frac{1}{N_i}\left[\sum_i \sigma_{\Phi CAL_{hij}}^2\right]^{1/2} \qquad \text{(check for bad bins)} \qquad \textbf{(EQ 7.12)}$$

# Part III
# A3 - Data Reduction

# CHAPTER 8

# A31 - Extract Measurement Parameters

## 8.1 A311 : Calculate intrinsic phase

### FUNCTIONAL DESCRIPTION:

The phase due to the mirror stepping is added to the other phase contributions to derive the total phase without atmospheric wind phase or to derive the intrinsic phase.

### INPUT:

The input parameters are:

- $\Phi_{V0_{hij}}$ - Phase due to projected velocity terms and zero wind phase. (from A2224).

- $\delta\phi_k(\lambda)$ - Phase step for measurement of mirror motion. (from CDB, one value for each wavelength and each phase step, approximately or ).

- $COAE_{hpq}$ - CDB off axis effect in CDB bins (from CDB). (see also function A222)

- $N_k$ - Number of mirror phase steps.

### OUTPUT:

The output parameters are:

- $\Phi_{hijk}$ - Total or intrinsic phase for each bin.

### HYPOTHESIS OF APPLICABILITY:

All the phase contributions are added together including the step due to mirror position. The off axis effect is computed for the step. The mirror phase step calculation is repeated here in order to reduce the catalogued data required in the first level of processing.

### CRITERIA OF APPLICABILITY:

Applied to all atmospheric doppler measurements.

### TRANSFORMATION EQUATION:

$$N_{BC} = \text{number of CDB bins in measurement bin.} \qquad \text{(EQ 8.1)}$$

$$MOAE_{hij} = \frac{1}{N_{BC}} \sum_p \sum_q COAE_{hpq} \qquad \forall (p, q) \text{ in } (i, j) \qquad \text{(EQ 8.2)}$$

$$\delta\phi_{hijk} = MOAE_{hij} \times \sum_{p=1}^{k} \delta\phi_p (\lambda) \quad \text{for } k = 1, N_k \qquad \text{(EQ 8.3)}$$

$$\Phi_{hijk} = \Phi_{V0_{hij}} + \delta\phi_{hijk} \qquad \text{(EQ 8.4)}$$

## 8.2 A312 : Calculate instrument visibility

### FUNCTIONAL DESCRIPTION:

The instrument visibility stored in the CDB is converted from the CDB bins to the measurement bins by averaging.

### INPUT:

The input parameters are:

- $U_{CDB_{hpq}}$ - Visibility from the CDB in CDB bins.

### OUTPUT:

The output parameters are:

- $U_{hij}$ - Visibility in measurement bins.

### HYPOTHESIS OF APPLICABILITY:

Assumes that the visibility measured in the infrequent calibrations and stored in the CDB can be averaged across the measurement bins to provide the instrument visibility appropriate for the measurement.

### CRITERIA OF APPLICABILITY:

Applied to all atmospheric doppler measurements.

### TRANSFORMATION EQUATION:

$$U_{hij} = \frac{1}{N_{BC}} \sum_p \sum_q U_{CDB_{hpq}} \qquad \forall (p, q) \text{ in } (i, j). \qquad \text{(EQ 8.5)}$$

where $N_{BC}$ is the number of CDB bins in one measurement bin as in A2121.

Note: A different bin combination may be required for the visibility. This will have to be confirmed during the instrument calibration.

## 8.3 A313: Calculate apparent quantity vectors. (J1,J2,J3)

### FUNCTIONAL DESCRIPTION:

The apparent quantity vectors J1, J2 and J3 may be obtained in one of two ways depending upon the measurement. Two functions A3131, the MLR fit, and A3132, the MPA fit, are described first. These routines are functions which are called by different parts of the software. They are important to understanding the calculation of the apparent vectors and so are included first. The MLR and MPA tools are called as needed in the remaining algorithms.

### INPUT:

- *NOWAVE* -Nowave limiting correlation coefficient value. (from CDB, typical value is 0.6)

- $U_{hij}$, $\Phi_{hijk}$, $I_{hijk}$, $Flags_{hijk}$ - Measurement data (as defined in following subsections).

### OUTPUT:

- $J1_{hi}$, $J2_{hi}$, $J3_{hi}$ - Column vectors of average column.

- $\sigma_{J1_{hi}}$, $\sigma_{J2_{hi}}$, $\sigma_{J3_{hi}}$ - Estimate of the standard deviation of the J vectors.

- $\rho_{wave, h}$ - Wave/nowave correlation coefficient.

### HYPOTHESIS OF APPLICABILITY:

It is assumed in all cases that the entire image is used to obtain a representative column or average column for that measurement. This entails defining a common altitude scale and average instrument parameters.

### CRITERIA OF APPLICABILITY:

### TRANSFORMATION EQUATIONS:

1. Call A3133

    - determine $\rho_{wave, h}$.

2. If $\rho_{wave, h} < NOWAVE$

    then Call A3134 else Call A3135

3. Call A3136

    - compute average altitudes and instrument parameters.

### 8.3.1 A3131: Multiple linear regression (MLR)

FUNCTIONAL DESCRIPTION:

A multiple linear regression method is used to fit the measured data to the generalized form of the Michelson equation. The data may be from either calibrations or measurements. A brief description of the Michelson equation with an optional polynomial pre-multiplier is given below. This formulation is weighted more strongly by the brightness variations and so averages wave structure in the intensity profile more than in the wind and temperature profiles.

**This function is divided into three parts. A31311 and A31312 initialize all the parameters needed for the actual MLR fit in A31313. Each of these three functions must be able to be called separately since the frequent calibration data extraction calls just A31313 while the meassurement data extraction calls all three.**

TRANSFORMATION EQUATION DESCRIPTION:

Systematic trends are introduced into the WINDII intensity data by wave motion and other atmopheric structures. Spatial variations from image to image will lead to errors in the apparent quantities if such trends are not accounted for in the analysis. The generalized form of the Michelson equation can be extended to include a linear or quadratic polynomial in place of the simple constant multiplier. This polynomial can then be used to model variations and reduce the error in the apparent quantity determination.

The measured intensity of a given bin at a given time may be written in the generalized form as:

$$I_p = A(t_p) \int_L E(z) \, [\, 1 + u_p V(z) \cos(\Phi_p + \phi(z)) \,] \, dl \qquad \text{(EQ 8.6)}$$

Where:

- $A(t_p)$ is one of

  $A(t_p) = a_0$ or

  $A(t_p) = a_0 + a_1 t_p$ or

  $A(t_p) = a_0 + a_1 t_p + a_2 t_p^2$

- $E(z)$ is the volume emission rate at altitude z.
- $u_p$ is the intrinsic visibility of a given bin.
- $V(z)$ is the line visibility corresponding to the atmospheric temperature at altitude z.

- $\Phi_p$ is the phase due to OPD, spacecraft velocity and mirror position.
- $\phi(z)$ is the phase due to the atmospheric wind.

and index $p = 1, 2, ..., n$. This assumes n data points were taken for the measurement.

If the cosine argument is expanded then equation 1 may be written as:

$$I_p = A t_p \int_L E z \, dl + A(t_p) u_p \cos \Phi_p \int_L E(z) V(z) \cos(\phi(z)) \, dl$$

$$-A(t_p) u_p \sin \Phi_p \int_L E(z) V(z) \sin(\phi(z)) \, dl \qquad \text{(EQ 8.7)}$$

$$I_p = A(t_p) J_1 + A(t_p) u_p \cos \Phi_p J_2 - A(t_p) u_p \sin \Phi_p J_3 \qquad \text{(EQ 8.8)}$$

The values $J_1$, $J_2$ and $J_3$ are the line integrals in equation 8.8 and they contain the atmospheric information we wish to recover.

For the case of $A(t_p) = a_0 + a_1 t_p + a_2 t_p^2$:

$$I_p = a_0 J_1 + a_0 J_2 (u_p \cos \Phi_p) t_p - a_0 J_s (u_p \sin \Phi_p)$$

$$+ a_1 J_1 t_p + a_1 J_2 (u_p \cos \Phi_p) t_p - a_1 J_3 (u_p \sin \Phi_p) t_p$$

$$+ a_2 J_1 t_p^2 + a_2 J_2 (u_p \cos \Phi_p) t_p^2 - a_2 J_3 (u_p \sin \Phi_p) t_p^2 \qquad \text{(EQ 8.9)}$$

This may be written as a linear system of equations

$$I_p = \sum_{q=0}^{N_{term}} b_q X_q (u_p, \Phi_p, t_p) \qquad \text{(EQ 8.10)}$$

where for $N_{term} = 8$ we have:

$$b_0 = a_0 J_1 \qquad X_0 = 1 \qquad \text{(EQ 8.11)}$$

$$b_1 = a_0 J_2 \qquad X_1 = (u_p \cos \Phi_p) \qquad \text{(EQ 8.12)}$$

$$b_2 = a_0 J_3 \qquad X_2 = -(u_p \sin \Phi_p) \qquad \text{(EQ 8.13)}$$

$$b_3 = a_1 J_1 \qquad X_3 = t_p \qquad \text{(EQ 8.14)}$$

$$b_4 = a_1 J_2 \qquad X_4 = (u_p \cos \Phi_p) t_p \qquad \text{(EQ 8.15)}$$

$$b_5 = a_1 J_3 \qquad X_5 = -(u_p \sin \Phi_p) t_p \qquad \text{(EQ 8.16)}$$

$$b_6 = a_2 J_1 \qquad X_6 = t_p^2 \qquad \text{(EQ 8.17)}$$

$$b_7 = a_2 J_2 \qquad X_7 = (u_p \cos \Phi_p) t_p^2 \qquad \text{(EQ 8.18)}$$

$$b_8 = a_2 J_3 \qquad X_8 = -(u_p \sin \Phi_p) t_p^2 \qquad \text{(EQ 8.19)}$$

For the case of a constant term fit: ($N_{term} = 2$)

$$J_1 = b_0 \qquad \text{(EQ 8.20)}$$

$$J_2 = b_1 = b_0 V \cos\phi \qquad \text{(EQ 8.21)}$$

$$J_3 = b_2 = b_0 V \sin\phi \qquad \text{(EQ 8.22)}$$

**REFERENCES:**

The Multiple Linear Regression Fitting routine described in A31312 and A31313 is a modified version of that described by Bevington on pages 164 to 186 in:

**Data Reduction and Error Analysis for the Physical Sciences**, Philip R. Bevington, 1969 (published by McGraw-Hill Book Company).

The modification is to define the fitting function to be in three independent variables in stead of just one. See A31313 below.

#### 8.3.1.1 A31311: Stack selected bins for fitting function

**FUNCTIONAL DESCRIPTION:**

The phase steps for a given bin are assigned sequentially to a vector for each bin in a given horizontal row. The *Flags* map defines which bins are good data and may be used in the fit.

**INPUT:**

The input parameters are:

- *nfov* - Field of View number 1 or 2.
- *rn* - Row Number of current horizontal row.
- $Flags_{hijk}$ - Include map of horizontal row. (must be obtained from level 1 processing)
- $N_j$ - Number of columns in a row.
- $N_k$ - Number of phase steps.
- $\Phi_{hijk}$ - total instrument phase or intrinsic phase.
- $I_{hijk}$ - correceted intensity for each bin and phase step.
- $\sigma^2_{hijk}$ - error estimate of corrected intensity.
- $U_{hij}$ - instrument visibility for each bin.
- *mode* - mode of weighting used in fitting procedure.

**OUTPUT:**

The output parameters are:

- $N_p$ - Number of data elements in the fit vector.
- $\phi^f_p$ - phase at step p in the fit vector.

- $t_p^f$ - stacking phase fit vector.

- $u_p^f$ - instrument visibility fit vector.

- $B_p^f$ - bin intensity fit vector (dependent coordinate).

- $\sigma_p$ - error estimate of bin intensity vector.

- $NOFIT$ - flag to indicate that there is not enough data to fit. ('1' means not enough data, '0' means sufficient data for fit)

## HYPOTHESIS OF APPLICABILITY:
Generally applicable.

## CRITERIA OF APPLICABILITY:
Applied to all doppler observation images. The number of data points which compose the fit is checked and if 30% of the data is missing then the data is flagged and no fit is carried out on the data.

## TRANSFORMATION EQUATION:

$h = nfov$

$i = rn$

$U_{hijk} = U_{hij}$ for $k = 1, N_k$   (visibility does not change with phase step)

$p = 0$
Do $j = 1, N_j$
    Do $k = 1, N_k$
        If $Flags_{hijk} = 1$ Then
            $p = p + 1$
            $t_p^f = \Phi_{hijk} + (j - 1) * 2\pi$
            $\phi_p^f = \Phi_{hijk}$
            $u_p^f = U_{hijk}$
            $B_p^f = I_{hijk}$
            If $mode = 0$ then $\sigma_p = 1$ else $\sigma_p = \sqrt{\sigma_{I_{hijk}}^2}$
        End If
    End Do
End Do

$N_p = p$

If $N_p < 0.7 \, N_j \, N_k$ then $NOFIT = 1$ Else $NOFIT = 0$

Figure 8.1 shows a sample of the stacking generated by this function. Vector $B_p^f$ is plotted against $t_p^f$, the stacking phase. In the case shown there are 8 phase steps reconstructed to appear as 8 fringes.

### 8.3.1.2  A31312: Initialize fitting parameters

#### FUNCTIONAL DESCRIPTION:
Initialize the parameters needed for the MLR fit. These are parameters used by the MLR routine to determine to mode of the fitting. (Note: these may change depending upon the input requirements of the MLR routine used)

#### INPUT:
The input parameters are:

*   *order* - Order of the MLR averaging polynomial (0,1 or 2).

#### OUTPUT:
The output paramteres are:

*   $N_{term}$ - number of function terms included in the fit.
*   $jterm_q$ - vector assigning the function terms in the fit. (depends on the MLR routine)

#### HYPOTHESIS OF APPLICABILITY:
Assumes the mode will be fixed in the operational software and will not be changed by run time parameters.

#### CRITERIA OF APPLICABILITY:
Applied to all calls to the fitting routine.

#### TRANSFORMATION EQUATION:

$$N_{term} = 2 + 3*order \qquad \text{(EQ 8.23)}$$

$$jterm_q = q \ \text{ for } \ q = 1, N_{term} \qquad \text{(EQ 8.24)}$$

Note: This algorithm (A31312) may be integrated with A31311. It has been kept separate since both the frequent phase calibration analysis and the atmospheric emission line analysis use the overall MLR routine in slightly different ways.

### 8.3.1.3  A31313: Multiple Linear Regression Fitting

#### FUNCTIONAL DESCRIPTION:
Multple Linear Regression (MLR) provides a least squares fit to a set of data using a basis set of functions which may be polynomials or may be more complex functions. Its advantage over straight forward matrix methods is that it requires the solution of one fewer simultaneous equation.

A standard multiple linear regression routine from a subroutine package like NAG may be used. The function definition may, however, need to be modified in such a standard

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for GRAPH

routine. The basis fitting functions, described below, require three (3) independent input variables as opposed to the usual one input variable. A single value is still returned for each set of inputs.

The MLR subroutine also calls a matrix inversion routine. The required routine will be in the math subroutine package.

### INPUT:

The input parameters are:

- $u_p^f$ - instrument visibility vector.

- $\phi_p^f$ - total phase vector (all sources except wind)

- $t_p^f$ - stacking phase (from A31311). ($u^f$, $\phi^f$ and $t^f$ are the independent variables for the fitting basis function)

- $B_p^f$ - bin intensity vector.

- $\sigma_p$ - standard deviation of bin intensity data.

- $N_p$ - number of points in the input data.

- $N_{term}$ - number of terms in the fit or number of coefficients.

- $jterm_q$ - vector assigning the order of the function terms in the fit. (depends on MLR routine)

- $mode$ - mode of the weighting used in the fit. (depends on MLR routine)

### OUTPUT:

The output parameters are:

- $yfit_p$ - vector with the calculated fit.

- $b_0$ - constant term fitting coefficient.

- $b_q$ - vector of fitting coefficients.

- $\sigma_{b_0}$ - standard deviation of b0.

- $\sigma_{b_q}$ - vector of standard deviation of bq.

- $r_q$ - linear correlation coefficients.

- $rmul$ - multiple linear correlation coefficient.

- $X^2$ - reduced Chi squared for the fit.

- $F$ - value of F test of fit.

### HYPOTHESIS OF APPLICABILITY:

A standard math library subroutine may be used to compute the Multiple Linear Regression fit, however, the function defining the basis vectors is not standard and requires 3 independent variables as inputs. The MLR routine used must be able to handle the basis function defined below.

## CRITERIA OF APPLICABILITY:

The number of data points, Np, must be greater than the number of terms in the fit or greater than $(N_{term} + 2)$ in order to obtain a solution. (eg. for constant term fit, order = 0, $N_p > 4$)

## TRANSFORMATION EQUATION:

This defines the basis functions used in the fitting procedure. Each call to this function returns only the value of one term of the basis functions. The *jterm* vector allows that different basis functions may be used for different calls to the routine. The sum of all terms times the fitting coefficients gives the estimate of the measured intensity.

The input parameters to the function are:

- $u_p^f$ - instrument visibility at point p.

- $\phi_p^f$ - total phase (all sources except wind) at point p.

- $t_p^f$ - stacking phase at point p.

- $p$ - current data point in input vector

- $q$ - current term in function.

- $jterm_q$ - vector assigning order of fitting functions.

The output of the function is:

- *wfctn* - value of the function for the qth term and the pth data point using the vector *jterm* to specify terms in the function.

The function requires three independent inputs in addition to the index values. The MLR routine used must be able to call this function with $u_p^f$, $\phi_p^f$ and $t_p^f$ defined.

Normally the function is called from within A31313. It may be called indepedently if the value of the fit is required for a given set of fitting coefficients.

Basis Functions:

The value $r = jterm_q$ is used as the index for the functions $X_r$ where:

$$X_0 = 1 \qquad\qquad \text{(EQ 8.25)}$$

$$X_1 = (u_p^f \cos\phi_p^f) \qquad\qquad \text{(EQ 8.26)}$$

$$X_2 = -(u_p^f \sin\phi_p^f) \qquad\qquad \text{(EQ 8.27)}$$

$$X_3 = t_p^f \qquad\qquad \text{(EQ 8.28)}$$

$$X_4 = (u_p^f \cos\phi_p^f) \, t_p^f \qquad\qquad \text{(EQ 8.29)}$$

$$X_5 = -(u_p^f \sin\phi_p^f)\, t_p^f \tag{EQ 8.30}$$

$$X_6 = t_p^2 \tag{EQ 8.31}$$

$$X_7 = (u_p^f \cos\phi_p^f)\, t_p^2 \tag{EQ 8.32}$$

$$X_8 = -(u_p^f \sin\phi_p^f)\, t_p^2 \tag{EQ 8.33}$$

Return $wfctn = X_r$ for the given $p$ and $q$.

### 8.3.2  A3132: Multiple point algorithm (MPA)

#### FUNCTIONAL DESCRIPTION:
A simple matrix method is used to compute $J1$, $J2$ and $J3$ for $I_0$ a constant.

#### INPUT:
The input parameters are:

- $N_p$ - Number of points in the fit vector.

- $u_p^f$ - Instrument visibility of each bin.

- $\phi_p^f$ - Intrinsic phase of each bin.

- $B_p^f$ - Corrected intensities.

- $\sigma_p$ - Error estimate for corrected intensities.

#### OUTPUT:
The output parameters are:

- $J1$, $J2$, $J3$ - J values for each bin.

- $\sigma_{J1}$, $\sigma_{J2}$, $\sigma_{J3}$ - estimate of standard deviation from fit.

#### HYPOTHESIS OF APPLICABILITY:
This routine may only be used when a constant term fit is used, that is when I0 is constant. For small matricies this method may be computationally faster than the MLR method, but is exactly equivalent to an *order* = 0 MLR fit.

#### CRITERIA OF APPLICABILITY:
This algorithm is designed to work with one fringe of data. Hence $N_p = 4$ or $N_p = 8$ are the only two values allowed for this routine.

#### TRANSFORMATION EQUATION:
The 4 or 8 intensities measured in a bin can be expressed as follows:

$$\begin{bmatrix} B_1^f \\ B_2^f \\ : \\ B_{N_\ell}^f \end{bmatrix} = \begin{bmatrix} 1 & u_1^f \cos\phi_1^f & -u_1^f \sin\phi_1^f \\ 1 & u_2^f \cos\phi_2^f & -u_2^f \sin\phi_2^f \\ : & : & : \\ 1 & u_{N_p}^f \cos\phi_{N_p}^f & -u_{N_p}^f \sin\phi_{N_\ell}^f \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix}$$

(EQ 8.34)

This is a matrix equation of the form $B = AJ$
Hence we solve for $J$ as:

$$J = (A^T \Sigma^{-1} A)^{-1} A^T \Sigma^{-1} B$$

where $\quad \Sigma_{n,n}^{-1} = \left(\dfrac{1}{\sigma_p^2}\right) \quad$ and $\quad \Sigma_{n,m}^{-1} = 0 \quad \forall\ n \neq m$

(EQ 8.35)

Hence $\quad \Sigma^{-1}$ is $N_p \times N_p$ and is diagonal

Compute the $\sigma_J$ values from diagonal elements of $(A^T \Sigma^{-1} A)^{-1}$. That is

$$\sigma_{J_n} = \left[ (A^T \Sigma^{-1} A)_{n,n}^{-1} \right]^{1/2} \text{ for } n = 1, 2, 3. = (A^T {}^{-1} A)^{-1}n,n \text{ for } n=1,2,3.$$

### 8.3.3 A3133: Calculate visibility correlation coefficient

#### FUNCTIONAL DESCRIPTION:
The visibility is calculated on a bin by bin basis using only the phase data for one bin (4 or 8 steps). The visibility is estimated for these bins from either the MLR or the MPA fit. The apparent temperature is then calculated and a correlation coefficient is calculated based on the whole image.

#### INPUT:
The input parameters are:

- $U_{hij}$ - Instrument visibility of each bin. (from A312)

- $\Phi_{hijk}$ - Intrinsic phase of each bin. (from A311)

- $I_{hijk}$ - Corrected intensities. (from level 1 file)

- $\sigma_{I_{hijk}}^2$ - Error estimate for corrected intensities. (from level 1 file)

- $Flags_{hijk}$ - Good/bad data flags for each bin. (from level 1)

- $\Delta_\lambda$ - OPD for measurement from CDB.

- $\lambda_0$ - Wavelength for measurement emission line.

- $N_i$ - Number of rows in measurment image.

- $N_j$ - Number of columns in measurement image.

- $N_k$ - Number of phase steps or images in measurement.

## OUTPUT:
The output parameters are:

- $J1_{hij}, J2_{hij}, J3_{hij}$ - J values for each bin.

- $\rho_{wave, h}$ - Wave / Nowave correlation coefficient.

## HYPOTHESIS OF APPLICABILITY:
The apparent temperature gives the best estimate of the presence of waves in the field of view. However, the intensity or wind phase may also be used in the same manner as the temperature. The visibility or temperature is used here. If the temperature is calculated for each bin then it is a function of row number, $i$, and column number, $j$. The row number corresponds to tangent altitude and if a wave is present then within a given column the variations due to the wave should correlate. The correlation coefficient is calculated at each bin and the average value is assigned to $\rho_{wave, h}$. Values of $\rho_{wave, h}$ less than 0.6 indicate that no wave is present in the measurement.

## CRITERIA OF APPLICABILITY:
None.

## TRANSFORMATION EQUATION:

Call A31331:
Compute the MPA or MLR fit to the data in each bin to derive values for $J1_{hij}$, $J2_{hij}$ and $J3_{hij}$. In the case of the MLR fit this is a constant term fit or $order = 0$ fit. Note that the correlation coefficient must be calculated independently for each field of view.

Call A31332:
Compute the apparent temperature for each bin. These temperatures are the data points used in the calculation of the correlation coefficient below.

Call A31333:
Compute the wave/nowave correlation coefficient.

### 8.3.3.1  A31331: Calculate J1, J2, J3 for each bin

#### FUNCTIONAL DESCRIPTION:
The apparent quantities are calculated on a bin by bin basis using only the phase data for one bin (4 or 8 steps). The visibility is estimated for these bins from either the MLR or the MPA fit.

#### INPUT:
The input parameters are:

- $U_{hij}$ - Instrument visibility of each bin.

- $\Phi_{hijk}$ - Intrinsic phase of each bin.

- $I_{hijk}$ - Corrected intensities.

- $\sigma^2_{I_{hijk}}$ - Error estimate for corrected intensities.

- $Flags_{hijk}$ - Good/bad data flags.

- $N_i$ - Number of rows in measurement image.

- $N_j$ - Number of columns in measurement image.

- $N_k$ - Number of phase steps or images in measurement.

## OUTPUT:
The output parameters are:

- $J1_{hij}, J2_{hij}, J3_{hij}$ - J values for each bin.

- $\sigma_{J1_{hij}}, \sigma_{J2_{hij}}, \sigma_{J3_{hij}}$ - Error estimate for J values.

- $V_{hij}$ - Visibility from fit for each bin.

- - Estimate of standard deviation in visibility.

- $Nflag_{hij}$ - Flag for each bin to indicate that there were enough data to calculate $J$ values. (1 for fit, 0 for nofit)

## HYPOTHESIS OF APPLICABILITY:
Assumes that the visibility can be obtained from a single bin.

## CRITERIA OF APPLICABILITY:
If the $Flags$ variable indicates that more than one of the bins in a measurement is bad then that bin should not be used in the determination of the correlation coefficient. Either the MPA or MLR fit may be used. The choice will be decided upon based on the efficiency of the algorithm and will then be fixed.

## TRANSFORMATION EQUATION:

$U_{hijk} = U_{hij}$ for $k = 1, N_k$ (visibility does not change with phase step)

If MLR fit then: set $order = 0$, $N_p = 8$, $mode = 1$ and Call A31312 For each $(h, i, j)$ execute the following 3 steps:

1. Set the fit vectors for each bin.
$p = 0$
Do $k = 1, N_k$
    If $Flags_{hijk} = 1$ Then
$$p = p + 1$$
$$\phi^f_p = \Phi_{hijk}$$
$$u^f_p = U_{hijk}$$
$$B^f_p = I_{hijk}$$
$$\sigma_p = \sqrt{\sigma^2_{I_{hijk}}}$$

End If
End Do
$N_p = p$

**2**. Fit the data.

**If** $(N_p \geq 4)$ then

$Nflag_{hij} = 1$

If MLR fit then: Call A31313,

On return set $J1_{hij} = b_0$, $J2_{hij} = b_1$, $J3_{hij} = b_2$ and

$\sigma_{J1_{hij}} = \sigma_{b_0}$, $\sigma_{J2_{hij}} = \sigma_{b_1}$, $\sigma_{J3_{hij}} = \sigma_{b_2}$

If MPA fit then: Call A3132,

On return set $J1_{hij} = J1$, $J2_{hij} = J2$, $J3_{hij} = J3$ and

$\sigma_{J1_{hij}} = \sigma_{J1}$, $\sigma_{J2_{hij}} = \sigma_{J2}$, $\sigma_{J3_{hij}} = \sigma_{J3}$

**Else**

$Nflag_{hij} = 0$

(Not enough data to do the fit for the given bin)

**3**. Compute the visibility.

**If** $(Nflag_{hij} = 1)$ then

Call A3141 to obtain $V_{hij}$ and $\sigma_{V_{hij}}$

**Else**

$V_{hij} = 1$ and $\sigma_{V_{hij}} = 1E + 30$

### 8.3.3.2   A31332: Calculate temperature from bin

#### FUNCTIONAL DESCRIPTION:

Compute the apparent temperature for each bin using A333. These temperatures are the data points used in the calculation of the correlation coefficient.

#### INPUT:

The input parameters are:

- $N_i$ - Number of rows in measurement image.

- $N_j$ - Number of columns in measurement image.

- $V_{hij}$ - Visibility from A31331.

- $\Delta_\lambda$ - OPD for the measurement from the CDB.

- $\lambda_0$ - Wavelength of the measurement emission line.

#### OUTPUT:

The output parameters are:

- $T_{hij}$ - Apparent temperature for bin.

### HYPOTHESIS OF APPLICABILITY:
None.

### CRITERIA OF APPLICABILITY:
None.

### TRANSFORMATION EQUATION:

Call A333 to compute the temperature. (Set $VTsw = 1$ in call to A333)

#### 8.3.3.3    A31333: Calculate temperature correlation coefficient

### FUNCTIONAL DESCRIPTION:
The apparent temperature is used to calulate a correlation coefficient based on the whole image.

### INPUT:
The input parameters are:

- $N_i$ - Number of rows in measurement image.

- $Nj$ - Number of columns in measurement image.

- $T_{hij}$ - Temperature from each bin.

- $Nflag_{hij}$ - fit flag from A31331.

### OUTPUT:
The output parameters are:

- $\rho_{wave, h}$ - Wave / Nowave correlation coefficient.

### HYPOTHESIS OF APPLICABILITY:
The apparent temperature gives the best estimate of the presence of waves in the field of view. If the temperature is calculated for each bin then it is a function of row number, $i$, and column number, $j$. The row number corresponds to tangent altitude and if a wave is present then within a given column the variations due to the wave should correlate. The correlation coefficient is calculated at each bin and the average value is assigned to .

### CRITERIA OF APPLICABILITY:
None.

### TRANSFORMATION EQUATION:

For a given row $i$ (in FOV $h$) we have $j = 1$  to  $Nj$ data points $T_{hij}$. For another row $p$ $(p \neq i)$  we have $j = 1$  to  $N_j$ data points $T_{hpj}$.

$$\text{Let } \bar{T}_{hi} = \frac{1}{N_j} \sum_{j=1}^{N_j} T_{hij} \quad \text{and} \qquad \text{(EQ 8.36)}$$

$$\bar{T}_{hp} = \frac{1}{N_j} \sum_{j=1}^{N_j} T_{hpj} \qquad \text{(EQ 8.37)}$$

also

$$\text{If } (T_{hij} = 0) \text{ Then } t_{hij} = 0 \text{ Else } t_{hij} = T_{hij} - \bar{T}_{hi} \qquad \text{(EQ 8.38)}$$

$$\text{If } (T_{hpj} = 0) \text{ Then } t_{hpj} = 0 \text{ Else } t_{hpj} = T_{hpj} - \bar{T}_{hp} \qquad \text{(EQ 8.39)}$$

The correlation coefficient then is (one per FOV)

for $i = 1, (N_i - 1)$ and $p = i + 1, N_i$

$$\rho_{hip} = \frac{\sum_j (t_{hij} t_{hpj})}{\sqrt{\sum_j (t_{hij}^2) \sum_j (t_{hpj}^2)}} \qquad \text{(EQ 8.40)}$$

If $\sum_j Nflag_{hij} > 0$ then check good rows

and the average of the absolute values provides the wave criteria

$$\rho_{wave,h} = \frac{2}{N_i(N_i - 1)} \sum_{i,p} |\rho_{hip}| \qquad \text{(EQ 8.41)}$$

### 8.3.4   A3134: Calculate apparent vector in No-Wave case

**FUNCTIONAL DESCRIPTION:**

 This function is called if the wave criteria from A3133 indicates that no waves are detected. In this case a constant term fit to the data is sufficient.

**INPUT:**

The input parameters are:

* - see below.

**OUTPUT:**

The output parameters are:

* - see below.

### 8.3.4.1 A31341: Compute row mean values of J1, J2 and J3

#### FUNCTIONAL DESCRIPTION:
This function is called if the wave criteria from A3133 indicates that no waves are detected. In this case the simple bin by bin values of J1, J2 and J3 are averaged.

#### INPUT:
The input parameters are:

- $J1_{hij}, J2_{hij}, J3_{hij}$ - J values for each bin.

- $\sigma_{J1_{hij}}, \sigma_{J2_{hij}}, \sigma_{J3_{hij}}$ - estimated standard deviations in the J values.

- $Nflag_{hij}$ - Fit flag from A31331.

#### OUTPUT:
The output parameters are:

- $\overline{J1}_{hi}, \overline{J2}_{hi}, \overline{J3}_{hi}$ - Row average values.

- $\overline{\sigma}_{J1_{hi}}, \overline{\sigma}_{J2_{hi}}, \overline{\sigma}_{J3_{hi}}$ - standard deviation of the mean.

#### HYPOTHESIS OF APPLICABILITY:
The average value and the standard deviation relative to the mean are computed.

#### CRITERIA OF APPLICABILITY:
None.

#### TRANSFORMATION EQUATION:

Set

$$NF_{hi} = \sum_j Nflag_{hij} \tag{EQ 8.42}$$

then compute averages

$$\overline{J1}_{hi} = \frac{1}{NF_{hi}} \sum_j J1_{hij} \times Nflag_{hij}, \quad \overline{\sigma}_{J1_{hi}} = \frac{1}{NF_{hi}} \left[ \sum_j \sigma^2_{J1_{hij}} \right]^{1/2} \times Nflag_{hij} \tag{EQ 8.43}$$

$$\overline{J2}_{hi} = \frac{1}{NF_{hi}} \sum_j J2_{hij} \times Nflag_{hij}, \quad \overline{\sigma}_{J2_{hi}} = \frac{1}{NF_{hi}} \left[ \sum_j \sigma^2_{J2_{hij}} \right]^{1/2} \times Nflag_{hij} \tag{EQ 8.44}$$

$$\overline{J3}_{hi} = \frac{1}{NF_{hi}} \sum_j J3_{hij} \times Nflag_{hij}, \quad \overline{\sigma}_{J3_{hi}} = \frac{1}{NF_{hi}} \left[ \sum_j \sigma^2_{J3_{hij}} \right]^{1/2} \times Nflag_{hij} \qquad \text{(EQ 8.45)}$$

### 8.3.4.2    A31342: Test for acceptable signal to noise

**FUNCTIONAL DESCRIPTION:**

Use the sigma values calculated in A31341 to determine which rows need to be refit using the full image and the MLR constant term fit.

**INPUT:**

The input parameters are:

- $\overline{J1}_{hi}, \overline{J2}_{hi}, \overline{J3}_{hi}$ - Row average values.

- $\overline{\sigma}_{J1_{hi}}, \overline{\sigma}_{J2_{hi}}, \overline{\sigma}_{J3_{hi}}$ - Standard deviations of the mean.

- *Jlimit* - The maximum relative deviation allowed for simple row averages. (from CDB)

**OUTPUT:**

The output parameters are:

- $J1_{hi}, J2_{hi}, J3_{hi}$ - Apparent quantity vectors for the average or representative column.

- $\sigma_{J1_{hi}}, \sigma_{J2_{hi}}, \sigma_{J3_{hi}}$ - Standard deviations for the average column.

**HYPOTHESIS OF APPLICABILITY:**

The relative variation is compared with the Jlimit value to determine if a new fit is to be done.

**CRITERIA OF APPLICABILITY:**

**TRANSFORMATION EQUATION:**

**If**

$$\frac{\overline{\sigma}_{J1_{hi}}}{\overline{J1}_{hi}} \le Jlimit \text{ and } \frac{\overline{\sigma}_{J2_{hi}}}{\overline{J2}_{hi}} \le Jlimit \text{ and } \frac{\overline{\sigma}_{J3_{hi}}}{\overline{J3}_{hi}} \le Jlimit \qquad \text{(EQ 8.46)}$$

**Then**

$$J1_{hi} = \overline{J1}_{hi} \quad J2_{hi} = \overline{J2}_{hi} \quad J3_{hi} = \overline{J3}_{hi} \qquad \text{(EQ 8.47)}$$

$$\sigma_{J1_{hi}} = \overline{\sigma}_{J1_{hi}} \quad \sigma_{J2_{hi}} = \overline{\sigma}_{J2_{hi}} \quad \sigma_{J3_{hi}} = \overline{\sigma}_{J3_{hi}} \qquad \text{(EQ 8.48)}$$

**Otherwise**

Refit the data using A3131 with constant term fit and *mode* = 0. Call A31311 for each

row $i$ and each FOV $h$. With $order = 0$ Call A31312 and A31313. Then the output values are defined as:

$$J1_{hi} = b_0 \quad J2_{hi} = b_1 \quad J3_{hi} = b_2 \tag{EQ 8.49}$$

$$\sigma_{J1_{hi}} = \sigma_{b_0} \quad \sigma_{J2_{hi}} = \sigma_{b_1} \quad \sigma_{J3_{hi}} = \sigma_{b_2} \tag{EQ 8.50}$$

### 8.3.5 A3135: Calculate apparent vector in Wave case

#### FUNCTIONAL DESCRIPTION:
This function is called if the wave criteria from A3133 indicates that waves are detected. In this case two possible solutions are given in A31351 and A31352.

#### INPUT:
The input parameters are:

- - see below.

#### OUTPUT:
The output parameters are:

- - see below.

#### HYPOTHESIS OF APPLICABILITY:

#### CRITERIA OF APPLICABILITY:

#### TRANSFORMATION EQUATION:

#### 8.3.5.1 A31351: Compute full MLR fit to image

#### FUNCTIONAL DESCRIPTION:
This is currently the only function defined to treat the wave case. The raw data from the images are sent to A3131 with $order = 2$ for a quadratic fit. The fitting coefficients returned are then used to compute the average $J$ vectors.

#### INPUT:
The input parameters are:

- $I_{hijk}$ - Corrected intensity data for the full image and for all phase steps.

- $\sigma^2_{I_{hijk}}$ - Error estimate for corrected intensity.

- $\Phi_{hijk}$ - Total phase for each bin.

- $U_{hij}$ - Instrument visibility for each bin.

- $Flags_{hijk}$ - Good/bad data flags for each bin.

#### OUTPUT:
The output parameters are:

- $J1_{hi}, J2_{hi}, J3_{hi}$ - J values based on MLR fit.

- $\sigma_{J1_{hi}}, \sigma_{J2_{hi}}, \sigma_{J3_{hi}}$ - Estimated standard deviation of the $J$ values.

### HYPOTHESIS OF APPLICABILITY:

Use the MLR fitting routine to return a fit and then obtain average values of J1, J2 and J3.

### CRITERIA OF APPLICABILITY:

None.

### TRANSFORMATION EQUATION:

Compute the MLR fit for each Field of View, $h$, and each row, $i$, as follows:

Call A31311 with $I_{hijk}$, $U_{hij}$, $\Phi_{hijk}$, $\sigma^2_{I_{hijk}}$ , $Flags_{hijk}$ and $mode = 1$ to stack the image data along a row.

If $NOFIT = 0$ then

    Call A31312 with $order = 2$.

    Call A31313 with the $b_0, b_1, \ldots, b_8$ and $\sigma_b$'s returned.

Use fit to give average value for $J1_{hi}$, $J2_{hi}$ and $J3_{hi}$ for each row in each field of view as follows:

The best fit value of intensity at a given point may be calculated from:

$$I_p = b_0 + b_1 (u_p \cos \Phi_p) - b_2 (u_p \sin \Phi_p)$$

$$+ b_3 t_p + b_4 (u_p \cos \Phi_p) t_p - b_5 (u_p \sin \Phi_p) t_p$$

$$+ b_6 t_p^2 + b_7 (u_p \cos \Phi_p) t_p^2 - b_8 (u_p \sin \Phi_p) t_p^2 \qquad \text{(EQ 8.51)}$$

The above equation may be rearranged by collecting all the terms in $t$:

$$I_p = b_0 + b_3 t_p + b_6 t_p^2 + (b_1 + b_4 t_p + b_7 t_p^2) u_p \cos \Phi_p - (b_2 + b_5 t_p + b_8 t_p^2) u_p \sin \Phi_p \qquad \text{(EQ 8.52)}$$

This is in a form that looks like the standard Michelson equation with:

$$J1_p = b_0 + b_3 t_p + b_6 t_p^2 \qquad \text{(EQ 8.53)}$$

$$J2_p = b_1 + b_4 t_p + b_7 t_p^2 \qquad \text{(EQ 8.54)}$$

$$J3_p = b_2 + b_5 t_p + b_8 t_p^2 \qquad \text{(EQ 8.55)}$$

We can thus calculate visibility and phase at each data point as:

$$V_p = \frac{\sqrt{J2_p^2 + J3_p^2}}{J1_p} \quad \text{range} \ [0, 1] \tag{EQ 8.56}$$

$$\phi_p = \tan^{-1}\left(\frac{J3_p}{J2_p}\right) \quad \text{range} \ [-\pi, \pi] \tag{EQ 8.57}$$

(See note on inverse tangent function in A3142.)

and the average values may be easily found:

$$J1_A = \frac{1}{N_p} \sum_{p=1}^{N_p} J1_p \tag{EQ 8.58}$$

$$V_A = \frac{1}{N_p} \sum_{p=1}^{N_p} V_p \tag{EQ 8.59}$$

$$\phi_A = \frac{1}{N_p} \sum_{p=1}^{N_p} \phi_p \tag{EQ 8.60}$$

with

$$J1_{hi} = J1_A \qquad \sigma_{J1_{hi}} = \sigma_{b_0} \tag{EQ 8.61}$$

$$J2_{hi} = J1_A V_A \cos\phi_A \qquad \sigma_{J1_{hi}} = \sigma_{b_1} \tag{EQ 8.62}$$

$$J3_{hi} = J1_A V_A \sin\phi_A \qquad \sigma_{J1_{hi}} = \sigma_{b_2} \tag{EQ 8.63}$$

Else (*NOFIT* = 1) Flag row to indicate that no fit was possible.

### 8.3.5.2  A31352: Alternate wave correction

#### FUNCTIONAL DESCRIPTION:
This function is currently undefined. Provisions should be made for possible future development.

#### INPUT:
The input parameters are:

* - must be the same as A31351.

#### OUTPUT:
The output parameters are:

* - must be the same as A31351.

**HYPOTHESIS OF APPLICABILITY:**

**CRITERIA OF APPLICABILITY:**

**TRANSFORMATION EQUATION:**

**8.3.5.3    A31352: Alternate wave correction**

**FUNCTIONAL DESCRIPTION:**
This function is currently undefined. Provisions should be made for possible future development.

**INPUT:**
The input parameters are:

*    - must be the same as A31351.

**OUTPUT:**
The output parameters are:

*    - must be the same as A31351.

**HYPOTHESIS OF APPLICABILITY:**

**CRITERIA OF APPLICABILITY:**

**TRANSFORMATION EQUATION:**

**8.3.6    A3136: Compute row averages for reference column**

**FUNCTIONAL DESCRIPTION:**
The simple numerical average of the values across a given row is determined.

**INPUT:**
The input parameters are:

*   $Z_{hijk}$ - Tangent altitude points for all bins.

*   $U_{hij}$ - Instrument visibility values for bins.

**OUTPUT:**
The output parameters are:

*   $\bar{Z}_{hi}$ - Average altitude profile.

*   $\bar{U}_{hi}$ - Average instrument visibility for row.

**HYPOTHESIS OF APPLICABILITY:**
Assumes that the instrument visibility can be averaged across a given row. The average altitude profile is over all columns in a row and over all the phase images. The value of

$\Delta_\lambda$ changes by about 1 part in 100,000 across a typical row and so does not need to be averaged.

**CRITERIA OF APPLICABILITY:**

**TRANSFORMATION EQUATION:**

$$\bar{Z}_{hi} = \frac{1}{N_j}\sum_j \left[\frac{1}{N_k}\sum_k Z_{hijk}\right] \qquad \text{(EQ 8.64)}$$

$$\bar{U}_{hi} = \frac{1}{N_j}\sum_j U_{hij} \qquad \text{(EQ 8.65)}$$

## 8.4 A314 - Derive apparent phase, visibility and amplitude

### 8.4.1 A3141 : Calculate visibility from J1,J2 and J3

**FUNCTIONAL DESCRIPTION:**
Simple arithmetic manipulation of the values to extract the apparent quantities is used.

**INPUT:**
The input parameters are:

* $J1_{hi}$ - Radiance term from apparent quantity determination.

* $J2_{hi}$ - I0 V () from apparent quantity determination.

* $J3_{hi}$ - I0 V () from apparent quantity determination.

* $\sigma_{J1_{hi}}, \sigma_{J2_{hi}}, \sigma_{J3_{hi}}$ - Standard deviation of J values.

**OUTPUT:**
The output parameters are:

* $I_{A_{hi}}$ - Apparent radiance.

* $V_{A_{hi}}$ - Apparent atmospheric line visibility.

* $\sigma_{VA_{hi}}$ - Standard deviation of visibility.

**HYPOTHESIS OF APPLICABILITY:**
Assumes $J1$, $J2$ and $J3$ have been derived by the fitting routines. The input values may be derived from atmospheric measurements or from calibration measurements.

**CRITERIA OF APPLICABILITY:**
This transformation is required for all data.

### TRANSFORMATION EQUATION:

$$I_{A_{hi}} = J1_{hi} \qquad \text{(EQ 8.66)}$$

$$V_{A_{hi}} = \frac{\sqrt{J2_{hi}^2 + J3_{hi}^2}}{J1_{hi}} \quad \text{range } [0, 1] \qquad \text{(EQ 8.67)}$$

$$\sigma_{VA_{hi}} = \frac{1}{J1_{hi}} \left[ \frac{\sigma_{J2_{hi}}^2 J2_{hi}^2 + \sigma_{J3_{hi}}^2 J3_{hi}^2}{J2_{hi}^2 + J3_{hi}^2} + \sigma_{J1_{hi}}^2 V_{A_{hi}}^2 \right]^{1/2} \qquad \text{(EQ 8.68)}$$

### 8.4.2   A3142 : Calculate phase from J2 and J3

#### FUNCTIONAL DESCRIPTION:
Simple arithmetic manipulation of the values to extract the apparent quantities is used.

#### INPUT:
The input parameters are:

- $J2_{hi}$ - $I_0 V\cos(\phi)$ from apparent quantity determination.

- $J3_{hi}$ - $I_0 V\sin(\phi)$ from apparent quantity determination.

- $\sigma_{J2_{hi}}, \sigma_{J3_{hi}}$ - Standard deviation of $J$ values.

#### OUTPUT:
The output parameters are:

- $\phi_{A_{hi}}$ - Apparent atmospheric line phase.

- $\sigma_{\phi_{hi}}$ - Standard deviation of phase.

#### HYPOTHESIS OF APPLICABILITY:
Assumes $J1$, $J2$ and $J3$ have been derived by the fitting routines. The input values may be derived from atmospheric measurements or from calibration measurements.

#### CRITERIA OF APPLICABILITY:
This transformation is required for all data.

#### TRANSFORMATION EQUATION:

$$\phi_{A_{hi}} = \tan^{-1}\left(\frac{J3_{hi}}{J2_{hi}}\right) \quad \text{range } [-\pi, \pi] \qquad \text{(EQ 8.69)}$$

$$\sigma_{\phi_{hi}} = \frac{\left[\sigma^2_{J2_{hi}} J3^2_{hi} + \sigma^2_{J3_{hi}} J2^2_{hi}\right]^{1/2}}{J2^2_{hi} + J3^2_{hi}}$$

(EQ 8.70)

Note: The proper inverse tangent function must be used inorder to ensure that the calculated value of phase will be in the correct range. That is both the sign and magnitude of $J3$ and $J2$ must be used to determine the angle and the quadrant for the angle.

### 8.4.3   A3143 : Calculate amplitude from J2 and J3

#### FUNCTIONAL DESCRIPTION:
Simple arithmetic manipulation of the values to extract the apparent quantities is used.

#### INPUT:
The input parameters are:

- $J2_{hi}$ - $I_0 V \cos(\phi)$ from apparent quantity determination.

- $J3_{hi}$ - $I_0 V \sin(\phi)$ from apparent quantity determination.

- $\sigma_{J2_{hi}}, \sigma_{J3_{hi}}$ - Standard deviation of $J$ values.

#### OUTPUT:
The output parameters are:

- $A_{A_{hi}}$ - Apparent atmospheric line amplitude.

- $\sigma_{A_{hi}}$ - Standard deviation of the line amplitude.

#### HYPOTHESIS OF APPLICABILITY:
Assumes $J1$, $J2$ and $J3$ have been derived by the fitting routines. The input values may be derived from atmospheric measurements or from calibration measurements. The amplitude is used by the rotational temperature algorithm.

#### CRITERIA OF APPLICABILITY:
This transformation applies only to OH and $O_2$ measurements.

#### TRANSFORMATION EQUATION:

$$A_{A_{hi}} = \frac{\sqrt{J2^2_{hi} + J3^2_{hi}}}{2}$$

(EQ 8.71)

$$\sigma_{A_{hi}} = \frac{1}{2} \left[\frac{\sigma^2_{J2_{hi}} J2^2_{hi} + \sigma^2_{J3_{hi}} J3^2_{hi}}{J2^2_{hi} + J3^2_{hi}}\right]^{1/2}$$

(EQ 8.72)

# CHAPTER 9

# A32 - Deconvolution of apparent quantities

**1.** Objective

The WINDII operational software can potentially provide profiles of doppler temperature, rotational temperature, and horizontal wind speed as a function of altitude. An intermediate step required for the determination of these profiles is the deconvolution of the apparent quantity arrays $J1_h$, $J2_h$, and $J3_h$ (in *rayleighs*). These arrays are described in the previous chapter and are provided as a function of tangent height. A tangent height is defined as the altitude of the point where the reference observation line-of-sight perpendicularly intersects an earth centered geodetic radial vector.

The altitude profiles resulting from the deconvolution process are volume emission rate, visibility, amplitude, and wind phase. The amplitude is defined here as one half the product of the volume emission rate with the visibility. Doppler temperatures and wind speeds are directly calculated from the corresponding visibilities and wind phases. The volume emission rate profile must be determined in order to derive the visibility profile. Rotational temperatures are derived by using the ratio of volume emission rates, or amplitudes, for two spectral lines associated with the same atmospheric constituents.

This chapter features a description of the general algorithm being used to determine the altitude profiles of volume emission rate $E_h$ $(photons \bullet cm^{-3} \bullet sec^{-1})$, and of visibility $V_h$ (dimensionless), or of amplitude $A_h$ $(photons \bullet cm^{-3} \bullet sec^{-1})$, and of a wind phase component $\phi_h$ (*radians*). The flexibility of this general algorithm should facilitate the application of any future improvements. The random error standard deviation estimate arrays $\sigma_E^h$ and $\sigma_V^h$, or $\sigma_A^h$, and $\sigma_\phi^h$, are provided as quality parameters. The above arrays

will be used in defining level 3 data. Noteworthy results or parameters, including those defined below, may be catalogued instead of being provided as level 3 data. The provided goodness of fit of the final solutions to each given apparent quantity array is characterized by its chi-square statistic over the chi-square statistic 95% confidence interval. In addition, independent parameters derived by fitting either one or two altitude dependent skewed gaussians to the volume emission rate profile may also be produced. These parameters are the maximum emission rates, the altitudes of maximum emission, the average half-widths, and the skewness factors. The above agenda is repeated separately for each of the two fields of view ($h$ equals 1 and 2).

**2.** Transformation Equations

Each apparent quantity can be interpreted as a line-of-sight integration through the atmosphere. Assuming a spherically symmetric atmosphere and discretizing the latter into homogeneous spherical layers (Figure A32.1) permits the three apparent quantity arrays to be related to the desired altitude profiles by means of the matrix equations

$$J1_h = LE_h \qquad\qquad \text{(EQ 9.1)}$$

$$J2_h = LE_c^c \qquad\qquad \text{(EQ 9.2)}$$

$$J3_h = LE_h^s \qquad\qquad \text{(EQ 9.3)}$$

where

$$\begin{bmatrix} E_{hi}^c \\ E_{hi}^s \end{bmatrix} = E_{hi} V_{hi} \begin{bmatrix} \cos(\phi_{hi}) \\ \sin(\phi_{hi}) \end{bmatrix} \qquad\qquad \text{(EQ 9.4)}$$

All variables shown in the first three equations denote one-dimensional vectors, except for the line-of-sight integration matrix $L$.

The homogeneous layers are set up such that the above systems are preferably even-determined but, as shall be described later, may occasionally be under-determined. Their boundaries are defined, whenever possible, to correspond to the tangent height boundaries of the actual three-dimensional fields-of-view. The length of each apparent quantity vector array must be greater or equal to four for the application of deconvolutions.

**3.** General Solution Method

The above transformation equations are not well suited to properly deal with measurement random error and under-determined systems. Consequently, these expressions are replaced by more appropriate general solution equations. The adopted general solution approach for each of the unknowns consists of the application of a Twomey technique with statistical weighting (Twomey, 1963 and 1977). This scheme falls under the category of linear-constrained least-squares methods. Statistical weighting is introduced to account for any unequal distribution of measurement random error level.

A common feature of the propogation of random errors from the measurements to the solutions is error amplification. In the absence of constraints and in the presence of significant measurement random error, a final solution profile will likely exhibit what will appear to be large random oscillations. The Twomey deconvolutions can ensure that any such oscillations will be damped. This damping will not increase the amount of information that may be extracted from the solutions. Instead, it will hopefully make the available minimal information more immediately clear and will simultaneously provide tools required by the user for the extraction of this information from the solution profiles.

The fitting of altitude dependent skewed gaussians to the volume emission rate profile has been introduced mainly for handling under-determined systems. The occurence and handling of under-determined systems is briefly described later as one of four special cases. The flexibility of the software will also allow for the application of this fitting scheme to volume emission rate profiles associated with even-determined systems.

The most commonly requested deconvolution solutions will be those of volume emission rates, visibilities, and wind phases. For OH or $O_2$, the amplitude profile may be calculated instead of the volume emission rates and the visibilities. Its derivation is described later. The general Twomey method equations for the volume emission rate profile $E_h$, the visibility profile $V_h$, and the wind phase profile $\phi_h$, are

$$\left(L^T S_{J1_h}^{-1} L + \gamma_E H_E\right) E_h = L^T S_{J1_h}^{-1} J1_h \qquad \text{(EQ 9.5)}$$

$$\left(\tilde{L}^T S_{J2_h}^{-1} \tilde{L} + \gamma_{V^c} H_{V^c}\right) V^c = \tilde{L}^T S_{J2_h}^{-1} J2' \qquad \text{(EQ 9.6)}$$

$$\left(\tilde{L}^T S_{J3_h}^{-1} \tilde{L} + \gamma_{V^s} H_{V^s}\right) V^s = \tilde{L}^T S_{J3_h}^{-1} J3' \qquad \text{(EQ 9.7)}$$

where

$$\tilde{L}_{i,j} = L_{i,j} E_{j_j} \qquad \text{(EQ 9.8)}$$

$$\begin{bmatrix} J2'_i \\ J3'_i \end{bmatrix} = \frac{(LE_h)_i}{J1_{hi}} \begin{bmatrix} J2_{hi} \\ J3_{hi} \end{bmatrix} \qquad \text{(EQ 9.9)}$$

$$\begin{bmatrix} V_i^c \\ V_i^s \end{bmatrix} = v_{hi} \begin{bmatrix} \cos(\phi_{hi}) \\ \sin(\phi_{hi}) \end{bmatrix} \qquad \text{(EQ 9.10)}$$

$S_{J1_h}$, $S_{J2_h}$, and , $S_{J3_h}$ are diagonal covariance matrix estimates, the $H$s are constraint matrices, and the $\gamma$s are non-negative weighting factors (lagrangian multipiers). It is

assumed that variances for $J2'$ and $J3'$ are the same as for $J2_h$ and $J3_h$, hence the use of $S_{J2_h}$ and $S_{J3_h}$.

The constraint matrices will typically denote the square of low order finite difference matrix operators. Each applied constraint matrix is selected from a set of choices consisting of pre-defined matrix types. This set may be expanded later. Some of the available types of constraint matrices will make use of specific reference profiles, or a priori information. The reference profiles can be based, for example, either on the previous solutions or on arrays specified in auxiliary files. For the volume emission rate profiles, previous solutions and profiles resulting from skewed gaussian fitting are two examples of a priori information.

The constraint matrices are used to limit the range of the corresponding solution gradients along the vertical. These constraint matrices do not typically restrict the absolute position of the solution profiles. The influence of these matrices on the final solution is dictated by the relative size of each weighting factor and of the elements of the direct inversion solution covariance matrix inverse, e.g. $L^T S_{J1_h}^{-1} L$.

The constraint matrices also ensure numerical stability of the solution equations. This aspect is especially crucial with regard to the second and third solution equations, this due to the definition of $\tilde{L}$. Another scheme applied for improving numerical stability, and simultaneuously reducing the size of the matrix equations, is the elimination of atmospheric layers with expected or predicted weak volume emission rates.

On the assumption that the remaining systematic phase error contribution is negligeable, the non-linearity of the above solution equations should not be of major concern since the wind phases being determined should typically be less than 0.1 *radian* in magnitude, implying that

$$V_i^c \approx V_{hi} \qquad \text{(EQ 9.11)}$$

$$V_i^s \approx V_{hi}\phi_{hi} \qquad \text{(EQ 9.12)}$$

The constraint matrices used for the deconvolutions are not rigorously based on statistics. Therefore, calculation of associated solution standard deviations is not possible. Instead, the random error standard deviations calculated for $E_h$, $E^c$, and $E^s$ are those associated with the initial transformation equations. For example, the volume emission rate standard deviations are

$$\sigma_{E_{hi}}^2 = \left( L^T S_{J1_h}^{-1} L \right)_{i, i}^{-1} \qquad \text{(EQ 9.13)}$$

Once these are available, the standard deviation estimates for $V_h$ and $\phi_h$ can be calculated.

The selected goodness-of-fit parameters are three chi-square statistic ratios. Each of these three parameters has been defined as the ratio of an apparent quantity chi-square statistic to the 95% confidence interval chi-square statistic. This limiting value is approximated by $N + 2\sqrt{2N}$ where $N$ is the length of the apparent quantity array. The chi-square statistic ratio for $J1_h$ is defined as

$$chi1_h = \frac{\left\| S_{J1_h}^{\frac{-1}{2}} (LE_h - J1_h) \right\|^2}{N + 2\sqrt{2N}}$$

**(EQ 9.14)**

The standard deviation estimates, the chi-square statistic ratios, the weighting factors, and an indication of the selected types of constraint matrices, will be essential for the interpretation of the final solution profiles. The standard deviation estimates of the final temperatures and winds will be provided as level 3 data. The other parameters are catalogued.

The general solution method described above requires the initialization and possible updating of some key parameters. At the moment these consist only of indices specifying the solution approach, weighting factors, indices specifying the selected constraint matrix types and reference profiles, and of parameters related to the skewed gaussian fits (Table A32.3). Since measurements are taken alternately for various emission lines, a set of these parameters must be available for each line. In addition, the weighting factor values, the indices, and parameters required for initializing the skewed gaussian fits are provided off-line.

**4.** Special Cases

Typically, three deconvolutions are done. These yield one volume emission rate profile, one visibility profile, and one wind phase component profile. However, depending on the spectral line being observed, a special treatment may be required. The following describes these special cases. Most of these cases can be handled using the tools made available by the general solution method.

**4.1** Case 1

Measurements at two OH spectral lines may be taken alternately for the purpose of deriving rotational temperatures. Therefore, it could occasionally be requested that, for the weaker of the two emission lines (at 734.1 nm), only the deconvolution for the volume emission rates be conducted. The time assigned to the measurement will be the average of the times for the OH(P13) and OH(P14) measurements.

**4.2** Case 2

Another option for OH measurements is the calculation of amplitudes and wind phases instead of volume emission rates, visibilities, and wind phases. This scenario is included in the event that the potentially large background contribution to the apparent quantities cannot be correctly estimated. On the assumption that the visibility profile would be the

same for both OH lines, the ratio of the amplitudes should be equal to the ratio of volume emission rates, thus providing rotational temperatures.

Since the $O_2$ background is not measured, the above scenario could also be applied to $O_2$ once deconvolutions for this emission are defined.

The general Twomey method equations used in this circumstance are

$$\left( L^T S_{J2_h}^{-1} L + \gamma_{A^c} H_{A^c} \right) A^c = L^T S_{J2_h}^{-1} J2_h \qquad \text{(EQ 9.15)}$$

where

$$\left( L^T S_{J3_h}^{-1} L + \gamma_{A^s} H_{A^s} \right) A^s = L^T S_{J3_h}^{-1} J3_h \qquad \text{(EQ 9.16)}$$

The routines used for the general solution method are also employed for case 2.

### 4.3 Case 3

The vertical $O(^1S)$ emission rate distribution typically consits of two major emission regions. The lower emission region is located in the mesosphere while the other is found mainly in the thermosphere. During night time, the vertical separation between the two emission regions can be as much as about 100 km. In addition, the mesospheric emission region is often narrower than the thermospheric emission region. Therefore, the corresponding two tangent height observation windows are viewed in sequence instead of simultaneously. Since the visibility and phase solutions in the mesosphere can be strongly dependent on the solutions in the thermosphere, the two sets of measurements must be combined so that they may be deconvolved as a single set of measurements. Whenever mesospheric observations are preceeded and followed by thermospheric observations, the two sets of thermospheric measurements are linearly interpolated along the vertical and as a function of measurement time. It is possible to make a thermospheric correction for day time measurements as well as for night time. The time assigned to the measurement will be the time of the mesospheric measurement.

### 4.4 Case 4

WINDII is limited to taking measurements at tangent heights below about 320 km. Thermospheric emission layers often extend above this altitude. This scenario pertains mainly to the $O(^1D)$ 630.0 nm emission but may also occasionally occur for the $O(^1S)$ 557.7 nm and the $O^+$ 732.0 nm day time emissions. Consequently, some assumption must be made regarding the emission region above the highest tangent height (i.e., above the inversion window). The adopted scheme consists of incorporating a modelled profile of volume emission rate as part of the constraint matrix $H_E$, and of assuming a constant visibility and phase for this upper region.

The volume emission model consists of either one or two altitude dependent skewed gaussian functions. The skewed gaussians are fitted to the volume emission rate profile produced by Twomey deconvolution. Only the layers inside the initial inversion window

are used in the fitting. The skewed gaussian fit provides an approximation to the altitude variation of the volume emission rate in the emission region above the inversion window for the following deconvolution. This approach is applied only if the maximum emission rate altitude is below the highest tangent point. Otherwise deconvolutions are not applied.

**5.** Algorithm Structure

The deconvolution algorithm is subdivided into three main sections. The first section (A321) treats the aspects relevant to the determination of the line-of-sight matrix. This is followed by the deconvolution for the volume emission rate profile (A322). Using these results, the visibility and phase profiles are determined in the third section (A323). The treatment of each special case is imbedded in the overall algorithm and is specified by means of indices. Whenever possible, the same routines are used for the various cases.

A list of input parameters provided by previous chapters is given in Table A32.1. The output parameters for possible use in A33 are listed in Table A32.2. Tables A32.3 and A32.4 specify the suggested Characterization Data Base parameters and parameters to be catalogued, both pertaining only to A32. The parameters found in these lists may be described in more detail in the various sections of this chapter.

**INPUT:**
(for steps 1 and 6 of the Transformation Equations)

- DECONVOLUTION SCENARIO INDICES

    - $idecon1$ - Index specifying desired deconvolution scenario for J1.
    (0 - no deconvolution, >0 deconvolution scenario) (from CDB)

    - $idecon2$ - Index specifying desired deconvolution scenario for J2 and J3.
    (0 - no deconvolutions, >0 deconvolution scenario) (from CDB)

- MEASUREMENT SCENARIO DEPENDENT INDICES (to be defined prior to A32)

    - $iflagdr$ - Index specifying desired solution parameters. One index is required for each measurement scenario. (-1 - no inversions, 0 - emission rates, visibilities and phases, 1 - emission rates, 2 - amplitudes and phases)

    - $itherm$ - Index specifying if the initial and the first and/or second additional arrays are present. This index identifies if $O(^1S)$ thermospheric measurements are available to complement the mesospheric measurements. (0 - Neither the first nor the second additional arrays are present, 1 - the first additional vector is available, 2 - the second additional vector is available, 3 - both additional vectors are available.)

- INPUT RELATED TO MOST RECENT DECONVOLUTIONS

    - $NP_{F_h}$ - Previous filter number.

    - $NP_h$ - Dimension of following arrays.

    - $z_{p_{hi}}$ - Altitude array for solution arrays. (km) (size: NPh)

    - $E_{p_{hi}}$ - Volume emission rate solution array. (size: NPh)

- **STANDARD ARRAYS/CONSTANTS**

  - $N_F$ - Current filter number.

  - $N_i$ - Dimension of arrays to .

  - $\bar{Z}_{hi}$ - Geodetic tangent height (or altitude) array. (size: Ni) (km) (from A3136)

  - $J1_{hi}$ - Apparent emission rate array. (rayleighs) (from A31342)

  - $J2_{hi}$ - Apparent quantity cosinusodal component array. (rayleighs)

  - $J3_{hi}$ - Apparent quantity sinusoidal component array. (rayleighs)

  - $\sigma_{J1_{hi}}$ - error standard deviation array. (from A31342)

  - $\sigma_{J2_{hi}}$ - error standard deviation array.

  - $\sigma_{J3_{hi}}$ - error standard deviation array.

- **FIRST SET OF NIGHT TIME O($^1$S) THERMOSPHERIC ARRAYS/CON-STANTS**

  - $N1_i$ - Dimension of arrays associated with the first and second thermospheric apparent quantity arrays. The following six arrays are of length $N1_i$.

  - $\sigma_{J11_{hi}}$ - J11 error standard deviation array.

  - $\sigma_{J21_{hi}}$ - J21 error standard deviation array.

  - $\sigma_{J31_{hi}}$ - J31 error standard deviation array.

- **SECOND SET OF NIGHT TIME O($^1$S) THERMOSPHERIC ARRAYS/CON-STANTS**

  - $\sigma_{J12_{hi}}$ - error standard deviation array.

  - $\sigma_{J22_{hi}}$ - error standard deviation array.

  - $\sigma_{J32_{hi}}$ - error standard deviation array.

### INTERMEDIATE:

(from step 1 and for use in A321 to A323)

- $NN_i$ - Dimension of arrays. (work variable)

- $zcc_i$ - Tangent height (altitude) array. (km) (work variable)

- $idec1_h$ - Index specifying if deconvolution of J1 will be attempted. (0 - no, >0 - yes (specifies deconvolution method))

- $idec2_h$ - Index specifying if deconvolution of J2 and J3 will be attempted. (0 - no, >0 - yes (deconvolution scenario))

- $E_{hi}$ - Initialized volume emission rate profile. $(photons \bullet cm^{-3} \bullet s^{-1})$ (size: $NN_i$)

- $A_{hi}$ - Initialized amplitude profile. $(photons \bullet cm^{-3} \bullet s^{-1})$ (size: $NN_i$)

- $\sigma_{E_{hi}}$ - Initialized volume emission rate standard deviation array. (size: $NN_i$)

- $\sigma_{A_{hi}}$ - Initialized amplitude standard deviation estimate array. (size: $NN_i$)

- $chi1_h$ - Initialized chi-square statistic ratio for J1.

- $V_{hi}$ - Initialized visibility profile. (size: $NN_i$)

- $\phi_{hi}$ - Initialized wind phase component array. (radians) (size: $NN_i$)

- $\sigma_{V_{hi}}$ - Initialized visibility deviation estimate array. (size: $NN_i$)

- $\sigma_{\phi_{hi}}$ - Initialized wind phase standard deviation estimate array. (size: $NN_i$)

- $chi2_h$ - Initialized chi-square statistic ratio for J2.

- $chi3_h$ - Initialized chi-square statistic ratio for J3.

- $J1_i^o$, or $J1_i^o$, $J2_i^o$ and $J3_i^o$ - Apparent quantity arrays. (work arrays) (size: $NN_i$)

- $S1_i^o$, or $S1_i^o$, $S2_i^o$, and $S3_i^o$ - Variance arrays for the above apparent quantities. (size: $NN_i$)

- $ntherm$ - Number of thermospheric measurements being used during the green line thermospheric/mesospheric coupling. Partially defined using itherm.

### OUTPUT:

(from steps 5, and 6 of the Transformation Equations section)

- $NM_{hi}$ - Dimension of all solution arrrays. (see Tables A32.2 and A32.4) ($NM_{1i}$ is for field-of-view 1 and $NM_{2i}$ is for field-of-view 2)

- $\bar{Z}_{f_{hi}}$ - Final altitude array. (size: $NM_{hi}$)

- $NP_{F_h}$ - Filter number to be used for future deconvolutions

- $NP_h$ - Dimension of following arrays to be used for future deconvolutions.

- $z_{p_{hi}}$ - Altitude array for current solution arrays. (km)

- $E_{p_{hi}}$ - Current volume emission rate solution array.

### HYPOTHESIS OF APPLICABILITY:

(1) All input indices have been appropriately defined.
(2) The apparent quantity arrays are available.
(3) The apparent quantity standard deviation arrays are available.
(4) This section is called once for each of the two image fields of view. (i.e. $h=1$ and 2)
(5) When green line measurements are provided, either 1, 2, or 3 sets of apparent quantities are provided.
(6) All the arrays for the $O(^1S)$ double thermospheric data have the same dimension.

### CRITERIA OF APPLICABILITY:

Deconvolutions are requested. ($iflagdr \geq 0$)

**TRANSFORMATION EQUATION:**

(1) The initialization of working arrays and constants. This step is described below.

(2) Call A321 - O($^1$S) data coupling, determination of the inversion window, addition of layers above the inversion window, and definition of the line-of-sight matrix. (3) Call A322 - Volume emission rate deconvolution.

(4) Call A323 - Visibility (or amplitude) and wind phase deconvolutions.

(5) Set the length $NM_{hi}$ and the altitude array of final solution profiles using

$$NM_{hi} = NN_i$$

and

$$\bar{Z}_{f_h} = zcc$$

(6) Store the filter number and some of the solution arrays. These may be used during the following call to A32.

$$NP_{F_h} = N_F$$

If $idec1_h > 0$

$$NP_h = NM_{hi}$$
$$z_{p_h} = \bar{Z}_{f_h}$$
$$E_{p_h} = E_h$$

Otherwise, set $NP_h = 0$, $z_{p_h} = 0$, and $E_{p_h} = 0..$

Steps (3) and (4) will be applied only if the results of the preceding step allow it.

Description of step 1:

(i) Initialization of deconvolution scenario work indices:

If $0 \le iflagdr < 2$, the index $idec1_h$ is initialized by setting it equal to $idecon1$. Otherwise, $idec1_h$ is set equal to zero. If $iflagdr \ne 1$, and $iflagdr \ne -1$, $idec2_h$ is set equal to $idecon2$ and, otherwise, it is set equal to zero.

(ii) Initialization of work arrays and constants:

The working arrays and constants are initialized using the corresponding INPUT arrays and constants, i.e.,

$$NN_i = N_i$$

$$\overline{zcc} = Z_h$$

$$J1^o = J1_h$$

$$S1^o = \sigma^2_{J1_h}$$

$$J2^o = J2_h \qquad \text{if } idec2_h > 0$$

$$J3^o = J3_h \qquad \text{if } idec2_h > 0$$

$$S2^o = \sigma^2_{J2_h} \qquad \text{if } idec2_h > 0$$

$$S3^o = \sigma^2_{J3_h} \qquad \text{if } idec2_h > 0$$

(iii) All apparent quantity error variances

(i.e.: $S1^o_i$, $\sigma^2_{11,hi}$, $\sigma_{12,hi}$, $S2^o_i$, $\sigma_{21,hi}$, $S3^o_i$, and $\sigma^2_{32,hi}$) with initial values smaller than 1.0 are set equal to this lower limit.

(iv) The solution arrays and constants are initialized:

$E_{hi}$, $A_{hi}$, and are set equal to 0.001, the $V_{hi}$ are set equal to 1.0, and , , , and are set equal to 1000 (or some other large default value) for $i = 1$ to $N_i$ (when $itherm = 0$) or to $N_i + NI_i$ (when $itherm > 0$).

Also $chi1_h$, $chi2_h$, and $chi3_h$ are set equal to 100 (or some other large default value).

If required ($itherm > 0$), the dimension of the solution arrays is specified by setting

$$NN_i = N_i + NI_i$$

(v) The constant $nitherm$ is set equal to zero. (for use later)

**Figure A32.1:**     Representative path length AB+CD for the line of sight of bin $i'$ through layer $j'$.



(Scale distorted)

- $x_i$ - Reference tangent height for line of sight i
- $z_j$ - Upper boundary altitude for atmospheric layer j
- Re - Earth radius

The variable definitions are those used in A3215.
The tangent height and altitude decrease as the subscript value increases.
The number of fields of view is the same as the number of layers.

**Table A32.2:**    A32 Output Available to A33.

| Current label | Description |
|---|---|
| $idec1_h$ | Index specifying if deconvolution for $E_h$ for successful |
| $idec2_h$ | Index specifying if deconvolution for $V_h$, or $A_h$, and $\phi_h$ was successful |
| $NM_{hi}$ | Dimension of following arrays (A32) |
| $\bar{Z}_{f_{hi}}$ | Altitude array ($km$) (A32) |
| $E_{hi}$ | Volume emission rate profile $(photons \bullet cm^{-3} \bullet s^{-1})$ (A3224) |
| $\sigma_{E_{hi}}$ | Standard deviation array $(photons \bullet cm^{-3} \bullet s^{-1})$ (A3226) |
| $A_{hi}$ | Amplitude profile $(photons \bullet cm^{-3} \bullet s^{-1})$ (A3234) |
| $\sigma_{A_{hi}}$ | Standard deviation array $(photons \bullet cm^{-3} \bullet s^{-1})$ (A3235) |
| $V_{hi}$ | Visibility array (A3234) |
| $\sigma_{V_{hi}}$ | Standard deviation array (A3235) |
| $\phi_{hi}$ | Wind phase component array ($radians$) (A3234) |
| $\sigma_{\phi_{hi}}$ | Standrad deviation array ($radians$) (A3235) |

**Table A32.3:**    Suggested Characterization Data Base Parameters (required for each scenario)

| Current label | Description | Number of elements |
|---|---|---|
| $idecon1$ | Deconvolution scenario index (A32) | 1 |
| $idecon2$ | Deconvolution scenario index (A32) | 1 |
| $MAT1$, $MAT2$, MAT22, $MAT3$, MAT33 | Constraint matrix selection indices (A3222) | 3 or 5 |
| $iref1$, $iref2$, $iref3$ | Reference profile indices (A3221 and A3233 | 3 |
| $\gamma_E$, $\gamma_2$, $\gamma_{22}$, $\gamma_3$, $\gamma_{33}$ | Weighting factors (A3223) | 3 or 5 |
| $Y_{k,l,m,n}$ | Initialization parameters defining skewed gaussian fit parameters (A3213) $k = 1,2,3$ for constant, $\sin^2$ and $\sin^4$ coefficients $l = 1$ for mesospheric $l = 2$ for thermospheric $m = 1$ for day $m = 2$ for night $n = 1$ for half width $n = 2$ for skewness | 3x2x2x2 |
| $itermax$ | Maximum number of iterations for skewed gaussian fitting (A3227) | 1 |

**Table A32.4:**     Suggested Parameters to be Catalogued

| Current label | Description | Number of elements |
|---|---|---|
| $iflagdr$ | Index for type of solutions (A32) | 1 |
| $itherm$ | Index for $O^1(S)$ data (A32) | 1 |
| $idec1_h$ | Final deconvolution scenario index | 1 (per FOV) |
| $idec2_h$ | Final deconvolution scenario index | 1 |
| $chi1_h$, $chi2_h$, $chi3_h$, $chi4_h$ | Chi-square statistic ratios (A3225 and A3227) | 4 |
| $MAT1$, $MAT2$, $MAT3$ | Constraint matrix selection indices (A3222) | 3 or 5 |
| $iref1$, $iref2$, $iref3$ | Reference profile indices (A3221 and A3233) | 3 |
| $\gamma_E$, $\gamma_2$, $\gamma_3$ | Weighting factors (A3223) | 3 or 5 |
| $X_{h,k,l}$ | Final skewed gaussian fit parameters (A3227) | 4x2 |
| $NM_{hi}$ | Dimension of following arrays (A32) | 1 |
| $E_h$ | Volume emission rate profile (A3224) | $NM_{hi}$ |
| $\sigma_{E_h}$ | Standard deviation profile (A3226) | $NM_{hi}$ |
| $\sigma^2_{X^c_h}$ | Variance profile (A3235) | $NM_{hi}$ |
| $\sigma^2_{X^d_h}$ | Variance profile (A3235) | $NM_{hi}$ |

## 9.1 A321: Line-of-sight geometry definition

### FUNCTIONAL DESCRIPTION:

**1.** Objective

The line-of-sight matrix, denoted as L in the A32 FUNCTIONAL DESCRIPTION, must be calculated such that deconvolutions may be applied. This line-of-sight matrix is required to transform arrays provided as a function of line of sight to corresponding arrays denoted as a function of altitude. The matrix elements can be determined once a few assumptions are established and a few important considerations are taken into account.

**2.** Assumptions

The assumptions pertaining to the atmosphere are that it can be treated as spherically symmetric, that it can be discretized into homogeneous layers with thicknesses approximately equivalent to the difference between tangent heights and, whenever possible, with altitude boundaries approximately equivalent to tangent height boundaries of individual bin fields-of-view. In addition, it is assumed that the measurement bin fields-of-view, which are finite, can be approximated by infinitessimal fields-of-view, i.e. by lines-of-sight. These assumptions are essential in order to minimize the work required in defining $L$. (Figure A32.1)

**3.** Special considerations

There are three important considerations, each requiring special attention. The first consists of combining thermospheric and mesospheric $O(^1S)$ measurements (Case 3 of the A32 FUNCTIONAL DESCRIPTION). The latter is required only for $O(^1S)$ data taken during night time. Typically, the mesospheric measurements are taken between two thermospheric measurements. The final profile consists of the mesospheric data coupled with thermospheric data obtained through interpolation in time and in tangent height.

The second consideration is the reduction of the apparent quantity arrays. All values of $J1_{hi}$ at the top of the inversion window which are smaller than the allowed lower limit of one standard deviation are defined as statistically unacceptable, and are therefore eliminated from the transformation equations. The corresponding layers, $J2_{hi}$, and $J3_{hi}$ are also discarded. Deconvolutions are not conducted if the resulting length of each apparent quantity array is less than four.

For thermospheric $O(^1D)$, $O(^1S)$ day time, and $O^+$ day time measurements, if the apparent emission rate associated with the highest tangent point is larger than this lower limit of one standard deviation, the deconvolution region may be extended upwards. (Case 4) The extension region is subdivided into homogeneous layers with constant thickness equal to that of the first underlying layer. A notable consequence of this extension is the change of the the transformation equations from even-determined to under-determined.

The determination of the upper boundary for this extended region relies on considering a skewed gaussian based analytical model defined either by a fit of the volume emission rate profile solved during the previous call to A32, or, after each filter change, by fit parameter estimates. The volume emission rate model is defined as

$$E(z, X) = \begin{cases} \sum_{l=1}^{2} X_{1,l} \, exp \left\{ -\ln(2) \left( \dfrac{z - X_{3,l}}{X_{2,l} + (\,_z - X_{3,l}) X_{4,l}} \right)^2 \right\} & \text{for } O(^1S) \\ X_{1,1} \, exp \left\{ -\ln(2) \left( \dfrac{z - X_{3,1}}{X_{2,1} + (\,_z - X_{3,1}) X_{4,1}} \right)^2 \right\} & \text{otherwise} \end{cases}$$

(EQ 9.17)

where the $X_{k,l}$ are the fit parameters and $z$ denotes altitude. (Thuillier, 1973). The fit parameters are also dependent on the FOV and on the filter, i.e. $h$ and $N_F$.

After each filter change, the fit parameters for the skewed gaussians must be estimated. The fit parameters are the maximum emission rate $X_1$, the altitude $X_3$ of maximum emission rate, the reference half-width $X_2$, and the skewness factor $X_4$ for each of either one (for $O(^1D)$ and $O^+$) or two (for $O(^1S)$) emission regions. The altitude of maximum volume emission rate is taken as equal to the tangent height of maximum apparent emission rate. The reference half-width and the skewness factor are initialized using the general function

$$X(\theta_h) = Y_1 + Y_2 \sin^2(\theta_h) + Y_3 \sin^4(\theta_h)$$

(EQ 9.18)

where denotes latitude and the array $Y$ is provided as input. The magnitude of the maximum emission rate for each emission region will be roughly approximated by dividing each corresponding maximum apparent emission rate, say $J1_{hi}$, by a line-of-sight segment of $2.5 L_{i,i}$.

The upper boundary altitude of the extended window is defined as the altitude corresponding to a volume emission rate value close to 5% of the emission rate peak. An upper limit altitude of 500 km is also imposed.

If the altitude of maximum thermospheric apparent emission rate is at the highest tangent point altitude, and if the estimated altitude of maximum volume emission rate in the thermosphere is greater or close to that same tangent height, then any deconvolution solutions that would be produced should be considered unreliable. Therefore, the currently requested deconvolutions will not be done.

**4.** Calculation of the line-of-sight matrix

Once the above considerations have been taken into account and the geodetic earth radius Re has been determined, the line-of-sight matrix can be calculated. The line-of-sight matrix element Li,j for each infinitessimal field-of-view i, and through each atmospheric layer j, is defined, using the Pythagorean theorem, as

$$L_{i,j} = 2c\left[\sqrt{(z_{j-1} - x_i)\ (z_{j-1} + x_i + 2R_e)} - \sqrt{(z_j - x_i)\ (z_j + x_i + 2R_e)}\right] \qquad \text{(EQ 9.19)}$$

where $R_e$ (km) is the geodetic earth radius, the $x_i$ (km) are the reference tangent heights for the infinitessimal fields-of-view, and $z_j$ (km) define layer altitude boundaries with $z_j < z_j - 1$ (A3215). The constant $c$ (=0.1) is the product of the km to cm and the

$photons \bullet cm^{-2} \bullet sec^{-1}$ to *rayleigh* conversion factors, respectively equal to $10^5$ and $10^{-6}$. The factor of 2 accounts for the line-of-sight crossing the atmospheric layer twice. The

matrix elements are in $\dfrac{rayleigh}{photons \bullet cm^{-3} \bullet sec^{-1}}$ . In Figure A32.1, the sum of segments

AB and CD corresponds to the matrix element with indices $(i',j')$.

### INPUT:

- $idec1_h$ - Index specifying requested deconvolution scenario for J1.
  (0 - no deconvolution, >0 - deconvolution scenario)

- $idec2_h$ - Index specifying requested deconvolution scenario for J2 and J3.
  (0 - no deconvolution, >0 - deconvolution scenario)

### HYPOTHESIS OF APPLICABILITY:

See assumptions provided in the FUNCTIONAL DESCRIPTION.

### CRITERIA OF APPLICABILITY:

Deconvolutions are requested. ($idec1_h > 0$ or $idec2_h > 0$)

### TRANSFORMATION EQUATION:

(1) Call A3211 - Combination of $O(^1S)$ night time thermospheric and mesospheric measurements.

(2) Call A3212 - Determine the reference geodetic earth radius.

(3) Call A3213 - Initialization of fit parameters for the volume emission rate profile. This is done for each atmospheric line, not for the background.

(4) Call A3214 - Determine the final deconvolution layers. The result is either the elimination or addition of atmospheric layers. When applicable, the number of added layers is provided.

(5) Call A3215 - Calculation of the line-of-sight matrix.

### 9.1.1 A3211: Combine thermospheric and mesospheric green line

### FUNCTIONAL DESCRIPTION:

For the night time green line ($O(^1S)$) case where thermospheric and mesospheric measurements are taken separately (i.e.: $itherm > 0$), these are combined to form single thermospheric/mesospheric arrays.

### INPUT:

- $NN_i$ - Dimension of arrays.

- $zcc_i$ - Tangent height (altitude) array. (km) (size: $NN_i$)

- $UT$ - Universal time of first image for J1. (sec)

- $J1_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)

- $J2_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)

- $J3_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)

- $S1_i^o$ - Variance array for J1^. (rayleighs$^2$)

- $S2_i^o$ - Variance array for $J2^o$.

- $S3_i^o$ - Variance array for $J3^o$.

- $itherm$ - Index specifying if the initial and the first and/or second additional apparent quantity arrays are present. (0 - Neither the first nor the second additional arrays are present, 1 - the first additional vector is availble, 2 - the second additional vector is available, 3 - both additional vectors are available.) This index is intended mainly for use with green line measurements.

- $ntherm$ - Number of thermospheric measurements being used during the green line thermospheric/mesospheric coupling.

- $N1_i$ - Dimension for first and second additional arrays for thermospheric night time $O(^1S)$ measurements.

- $\overline{Z1}_{hi}$ - First geodetic tangent height (altitude) array for thermospheric night time $O^1S$ measurements. (for J11) (size: $N1_i$)

- $UT1$ - First additional universal time (for J11)

- $J11_{hi}$ - First additonal apparent quantity array. First set of thermospheric night time $O(^1S)$ measurements. (rayleighs) (size: $N1_i$) (Taken before $J1$ in terms of measurement time. $itherm \geq 1$)

- $J21_{hi}$ - First additional apparent quantity array. First set of thermospheric night time $O(^1S)$ measurements ($itherm \geq 1$) (rayleighs) (size: N1i)

- J31hi - First additional apparent quantity array. First set of thermospheric night time $O(^1S)$ measurements ($itherm \geq 1$) (rayleighs) (size: $N1_i$)

- $\sigma_{J11_{hi}}$ - J11 error standard deviation array.

- $\sigma_{J21_{hi}}$ - J21 error standard deviation array.

- $\sigma_{J31_{hi}}$ - J31 error standard deviation array.

- $\overline{Z2}_{hi}$ - Second additional geodetic tangent height (altitude) array. (for $J12$) (size: $N1_i$)

- $UT2$ - Second additional universal time (for $J12$)
- $J12_{hi}$ - Second additonal apparent quantity array. Second set of thermospheric night time $O(^1S)$ measurements. (rayleighs) (size: $N1_{i)}$ (Taken after $J1$ in terms of measuremt time. $itherm \geq 2$)
- $J22_{hi}$ - Second additional apparent quantity array. Second set of thermospheric night time $O(^1S)$ measurements. (rayleighs) (size: $N1_i$) ($itherm \geq 2$)
- $J32_{hi}$ - Second additional apparent quantity array. Second set of thermospheric night time $O(^1S)$ measurements. ($itherm \geq 2$) (rayleighs) (size: $N1_i$)
- $\sigma_{J12_{hi}}$ - J12 error standard deviations.
- $\sigma_{J22_{hi}}$ - J22 error standard deviations.
- $\sigma_{J32_{hi}}$ - J32 error standard deviations.

## OUTPUT:

- $NN_i$ - Dimension of arrays.
- $zcc_i$ - Tangent height (altitude) array. (km) (size: $NN_i$)
- $NN_i$ - Dimension of standard arrays.
- $J1_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)
- $J2_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)
- $J3_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)
- $S1_i^o$ - Variance array for $J1^o$. (rayleighs$^2$)
- $S2_i^o$ - Variance array for $J2^o$.
- $S3_i^o$ - Variance array for $J3^o$.
- $ntherm$ - Number of thermospheric measurements being used during the green line thermospheric/mesospheric coupling.

## HYPOTHESIS OF APPLICABILITY:

1) The dimension parameter $NN_i$, the array $zcc$ and the arrays with superscript $^o$ have been properly initialized prior to the use of this algorithm.
2) The arrays for the two sets of thermospheric data have the same dimension (i.e.: $N1_i$)

## CRITERIA OF APPLICABILITY:

The combination of measurements is requested (i.e.: $itherm > 0$).

### TRANSFORMATION EQUATION:

(1) If there are two sets of thermospheric arrays (i.e.: *itherm* = 3) then the altitudes $\overline{Z1}_{hi}$ and $\overline{Z2}_{hi}$ are linearly interpolated in time (using *UT*, *UT*1 and *UT*2) for each *i*. The result is a common altitude profile Z3. This simple interpolation will preserve to some extent the field of view per bin implemented in the matrix line-of-sight calculation.

The apparent quantity arrays (i.e.: *J*11 and *J*12, *J*21 and *J*22, and *J*31 and *J*32) are then linearly interpolated along the vertical and in time (2-dimensional linear interpolation (i.e.: Bilinear interpolation)) to correspond to the derived common altitude profile and to *UT*. Each interpolated final apparent quantity value will have been determined by the interpolation of four of the intial apparent quantities. Special attention must be given to the determination of the first and last values in the interpolation arrays.

The final three variance arrays for the thermospheric apparent quantities are determined by properly summing the appropriate variances. E.g.: If

$$J13_i^o = a_i J11_{h,j} + b_i J11_{h,j+1} + c_i J12_{h,k} + d_i J12_{h,k+1} \qquad \textbf{(EQ 9.20)}$$

where $J13_i^o$ is the interpolated value and $a_i$, $b_i$, $c_i$ and $d_i$ are constants resulting from the bilinear interpolation, then the corresponding variance $S13_i^o$ is given by

$$S13_i^o = a_i^2 \sigma_{J11_{h,j}}^2 + b_i^2 \sigma_{J11_{h,j+1}}^2 + c_i^2 \sigma_{J12_{h,k}}^2 + d_i^2 \sigma_{J2_{h,k+1}}^2 \qquad \textbf{(EQ 9.21)}$$

(2) If there is only one set of thermospheric data, then no interpolation is possible. Consequently, this appropriate thermospheric data will be coupled with the mesospheric data without interpolation. (e.g.: If *itherm* = 2 then Z3 is set equal to $\overline{Z2}_h$)

(3) This step is done in order to eliminate overlapping of the field of views of the bins at the mesosphere/thermosphere interface. If $Z3_1$ is less than (i.e.: below) $zcc_1$ then no thermospheric data is included in the final arrays. Otherwise, all thermospheric related array elements associated with a $Z3_i$ ($i > 1$) where $2 \bullet Z3_i - Z3_{i-1}$ is smaller than zcc1 are not included in the final single thermospheric/mesospheric arrays.

(4) The single thermospheric/mesospheric arrays are defined.

$$zcc = [Z3, zcc]^T$$
$$J1^o = [J13^o, J1^o]^T$$
$$S1^o = [S13^o, S1^o]^T$$

If $idec1_h > 0$

$$J2^o = [J23^o, J2^o]^T$$

$$J3^o = [J33^o, J3^o]^T$$

$$S2^o = [S23^o, S2^o]^T$$

$$S3^o = [S33^o, S3^o]^T$$

Note that all the arrays in the input/output lists given above must be included in the thermospheric/mesospheric coupling.

(5) The final size of the measurement window $NN_i$ is re-set as equal to $NN_i + N1_i$ when there is no overlapping between the thermospheric and mesospheric windows. Otherwise, $NN_i$ is re-set equal to $NN_i + N1_i - (N1_i +1 - j)$ where thermospheric measurements $j$ to $N1_i$ have been eliminated.

(6) The number of thermospheric measurements at the top of the resulting arrays is stored in *ntherm* ($0 \le ntherm \le N1_i$).

### 9.1.2  A3212: Determine the reference geodetic earth radius

#### FUNCTIONAL DESCRIPTION:
The reference geodetic earth radius is determined.

#### INPUT:
* $\vartheta_{hi}$ - Reference tangent point geodetic latitude. (degrees)

#### OUTPUT:
* $R_e$ - Reference geodetic earth radius. (km)

* $\theta_h$ - Average reference tangent point latitude. (to be used in A3213)

#### HYPOTHESIS OF APPLICABILITY:
The planet Earth is approximated to an oblate spheroid. However, local sphericity is assumed in calculating the line-of-sight matrix. Consequently, the geodetic earth radius must be calculated (as opposed to the goecentric radius).

The provided geodetic latitude is in the interval [-90,90].

#### CRITERIA OF APPLICABILITY:
None

#### TRANSFORMATION EQUATION:
The Orbit/Attitude (OA) service routine which calculates the geodetic earth radius requires the goecentric latitude as input. However, the available latitude is geodetic. Therefore, the latter must be converted to geocentric latitude (i.e.:     ). This is done by applying the equations

$$\theta_h = \frac{1}{N_i} \sum_i \vartheta_{hi} \qquad \text{(EQ 9.22)}$$

$$\theta_c = \arctan \left( (1 - F)^2 \tan\theta_h \right) \qquad \text{(EQ 9.23)}$$

with $F = 1/298.257$ when $\theta_h$ is not equal to either -90 or 90 degrees. Otherwise $\theta_c$ is set equal to $\theta_h$. The above expression was taken from the Orbit/Attitude service routine OA_EARTH_RAD.

The geodetic earth radius is then calculated by a call to the above OA service routine, i.e.:

CALL OA_EARTH_RAD $(\theta_c, \theta_h, R_e, s)$

If the status index $s$ equals UOAS_GEOCLATINC then the radius could not be calculated (see Orbit/Attitude service routine documentation for information on UOAS_-GEOCLATINC). In this case the geodetic radius is set equal to the geocentric earth radius.

The geocentric earth radius $R_{ce}$ is determined by

$$R_{ce} = \sqrt{\left( \frac{r_e^2 \bullet r_p^2}{(r_e \sin(\theta_c))^2 + (r_p \cos(\theta_c))^2} \right)}$$

where $r_e$=6378.160 km and $r_p$=6356.775 km.

### 9.1.3  A3213: Initialization of skewed gaussian fit parameters

#### FUNCTIONAL DESCRIPTION:

Skewed gaussian fit parameters are initialized when skewed gaussian fitting of the volume emission rate profile is desired and when the filter has just been changed (i.e. $NP_{F_h} \neq N_F$)

The fit parameters are the maximum emission rate $X_1$, the altitude $X_3$ of maximum emission rate, the reference half-width $X_2$, and the skewness factor $X_4$ for each of either one or two (for O($^1$S)) emission regions.

#### INPUT:

- $\theta_h$ - Reference tangent height geodetic latitude from A3212. (degrees)
- $R_e$ - Reference geodetic earth radius. (km)
- *aperture* - Index specifying whether the day or the night aperture is used.

- $Y_{k,l,m,n}$ - Coefficients used in initializing the skewed gaussian fitting half-widths ($n$=1) and skewness parameters ($n$=2) (size: 3x2x2x2) (from CDB)

- $NP_{F_h}$ - Filter number for previous deconvolution period.

- $N_F$ - Current filter number.

- $itermax$ - Maximum number of iterations for skewed gaussian fitting. (0 - fitting is not allowed, > 0 - maximum number of iterations, fitting is attempted only when layers have been added above the inversion window, < 0 - its absolute value is the maximum number of iterations, fitting is attempted at all times) (from CDB)

- $idec1_h$ - Index specifying if deconvolution scenario for J1 was successful. (0 - no deconvolutions, >0 - deconvolution scenario)

- $NN_i$ - Dimension of arrays.

- $J1^o_{hi}$ - Radiance array. (rayleighs) (size: $NN_i$)

- $zcc_i$ - Tangent height array. (km) (size: $NN_i$)

### OUTPUT:

- $chi4_h$ - Initilization value for chi-square statistic ratio for Eh with respect to the skewed gaussian fit solution.

- $X_{h,k,l}$ - Initial skewed gaussian fit parameters. (size: 4x2 for given h value)

### HYPOTHESIS OF APPLICABILITY:
None.

### CRITERIA OF APPLICABILITY:

The deconvolution of $J1_h$ is requested ($idec1_h > 0$) and fitting is allowed ($itermax \neq 0$). Also the filter has just been changed (i.e. $NP_{F_h} \neq N_F$) and the new filter is for a different species (i.e. initialization is not done during subsequent switching of the OH filters, i.e. $N_F = 5$).

### TRANSFORMATION EQUATION:

The initial fit parameter values are defined. For O($^1$S) measurements (i.e. $N_F = 2$), fit parameters must be estimated for both the mesospheric and thermospheric region skewed gaussians (i.e. $l = 1$ to 2). Otherwise, only one set of fit parameters are initialized (i.e. $l = 1$ only).

For O($^1$S), the thermospheric fit parameters (i.e. $l = 2$) are derived from $J1^o_h$ and zcc for tangent heights above 140 km. The mesospheric fit parameters (i.e. $l = 1$) are derived from the measurements for tangent heights below 140 km. If $zcc_1$ is less than 140 km, $X_{h,1,2}$ and $X_{h,3,2}$ can be set equal to zero and $zcc_1$, respectively. Otherwise, if $zcc_{NN_i}$ is greater than 140 km, $X_{h,1,1}$ and $X_{h,3,1}$ are set equal to zero and $zcc_{NN_i}$, respectively.

The altitude $X_{h,1,1}$ of maximum volume emission rate is taken as equal to the tangent height of maximum apparent emission rate.

The magnitude of the maximum volume emission rate for each emission region will be roughly approximated by dividing each corresponding maximum apparent emission rate, say $J1_{hi}$, by a line-of-sight segment of $2.5L$ with $L$ defined using the Pythagorean theorem (see A3215)

$$L = 0.2\sqrt{(zcc_{i-1} - x)\,(zcc_{i-1} + x + 2R_e)}$$

where

$$x = \frac{1}{2}(zcc_{i-1} + zcc_i)$$

The reference half-width $X_{h,2,1}$ and the skewness factor $X_{h,4,1}$ are initialized using

$$X_{h,2,l} = Y_{1,l,m,1} + Y_{2,l,m,1}\sin^2(\theta_h) + Y_{3,l,m,1}\sin^4(\theta_h)$$

and

$$X_{h,4,l} = Y_{1,l,m,2} + Y_{2,l,m,2}\sin^2(\theta_h) + Y_{3,l,m,2}\sin^4(\theta_h)$$

for the day time or night time apertures respectively.

The constant $chi4_h$ is initialized as equal to 0.5 (or any value between 0 and 1). This constant will be required in A32143 and/or A322.

### 9.1.4   A3214: Determine deconvolution window

#### FUNCTIONAL DESCRIPTION:
Either eliminate inversion window layers or add layers above inversion window whenever required.

#### HYPOTHESIS OF APPLICABILITY:
None

#### CRITERIA OF APPLICABILITY:
None

#### TRANSFORMATION EQUATION:

(1) A32141 - Apply criteria for window modification. (Define *index*.)

(2) A32142 - Reduce window by eliminating small apparent quantities. (Applied only when *index* = 1.)

(3) A32143 - Add layers above inversion window. (Applied only when *index* = 2.)

### 9.1.4.1 A32141: Apply window modification criteria

#### FUNCTIONAL DESCRIPTION:
Set the index that specifies if and how the inversion window or the set of atmospheric layers will be changed.

#### INPUT:
- $NN_i$ - Dimension of standard arrays.

- $J1_i^o$ - Apparent quantity array. (rayleighs) (size: $NN_i$)

- $S1_i^o$ - Variance array for $J1^o$. (rayleighs$^2$)

- $N_F$ - Filter number (from A11)

- $\rho_{wave,h}$ - Wave/No-wave correlation coefficient. (from A3133) (not currently used)

- $idec1_h$ - Index specifying requested deconvolution scenario for $J1$. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $idec2_h$ - Index specifying requested deconvolution scenario for J2 and J3. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

#### OUTPUT:
- $NA$ - Initialized number of layers above inversion window.

- $J_{max}$ - Maximum $J1_i^o$.

- $i_{J_{max}}$ - Layer index of $J_{max}$.

- $idec1_h$ - Index specifying requested deconvolution scenario for J1. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $idec2_h$ - Index specifying requested deconvolution scenario for J2 and J3. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- *index* - Index specifying how solution window size is to be changed. (0 - No change, 1 - Reduction, 2 - Add layers)

#### HYPOTHESIS OF APPLICABILITY:
None

#### CRITERIA OF APPLICABILITY:
None

#### TRANSFORMATION EQUATION:

(1) Initialize index as equal to zero and $NA$ as equal to zero.

(2) If the initial size of the vector $J1^o$ is less than four (ie.: if $(NN_i \leq 4)$) then indices are set to indicate that no deconvolutions will be applied (i.e.: set $idec1_h = 0$, $idec2_h = 0$, and $index = 0$). Therefore we can escape from A32. (Note: A criterion to account for the presence of waves might be added later (e.g.: If $(\rho_{wave,h} > 0.6)$ then waves are present and inversions will not be done.).)

(3) Determine the maximum value $(J_{max})$ in the array $J1^o$. Also store the correspdong layer index $(i_{J_{max}})$. Both variables may be used in A32142 or A32143.

(4) Apply the window reduction criterion. Use the minimum of $\sqrt{S1^o_1}$ and one percent of the maximum value as the lower limit for $J1^o_1$ that is applied on the top side of the array.

If $((idec1_h.ne.0.or.idec2_h.ne.0).and.J1^o_1.lt.\min(\sqrt{S1^o_1}, 0.01J_{max}))$ $index = 1$

(5) Apply the window extension criterion. Extension of the window is possible only for the green line ($O(^1S)$, $N_F = 2$), the red line ($O(^1D)$, $N_F = 3$), and the 732 nm line ($O^+$, $N_F = 4$).

If $(idec1_h.ne.0.and.index.eq.0.and.zcc_1.gt.250.and.(N_F.ge.2.and.N_F.le.4))$ $index=2$

### 9.1.4.2 A32142: Reduce inversion window upper boundary

#### FUNCTIONAL DESCRIPTION:
The highest atmospheric layers (and apparent quantities associated with the highest tangent heights) corresponding to weak intensities are eliminated. No less than four layers must remain for applying deconvolutions.

#### INPUT:

- $J_{max}$ - Maximum $J1^o_i$.

- $NN_i$ - Dimension of standard arrays.

- $index$ - Index specifying how solution window size is to be changed. (0 - No change, 1 - Reduction, 2 - Addition of layers)

- $zcc_i$ - Tangent height (altitude) array. (km) (size: $NN_i$)

- $J1^o_i$ - Apparent emission rate array. (rayleighs) (size: $NN_i$)

- $J2^o_i$ - Apparent quantity (cosinusoidal component) array. (rayleighs) (size: $NN_i$)

- $J3^o_i$ - Apparent quantity (sinusoidal component) array. (rayleighs) (size: $NN_i$)

- $S1^o_i$ - Variance array for $J1^o$. (rayleighs$^2$)

- $S2^o_i$ - Variance array for $J2^o$.

- $S3_i^o$ - Variance array for $J3^o$.

- $ntherm$ - Number of thermospheric measurements being used during the green line thermospheric/mesospheric coupling.

- $idec1_h$ - Index specifying requested deconvolution scenario for J1. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $idec2_h$ - Index specifying requested deconvolution scenario for J2 and J3. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

## OUTPUT:

- $NN_i$ - Dimension of standard arrays.

- $zcc_i$ - Tangent height (altitude) array. (km) (size: $NN_i$)

- $zcc_i$ - Altitude array. (work array) (size: $NN_i$)

- $J1_i^o$ - Apparent emission rate array. (rayleighs) (size: $NN_i$)

- $J2_i^o$ - Apparent quantity (cosinusoidal component) array. (rayleighs) (size: $NN_i$)

- $J3_i^o$ - Apparent quantity (sinusoidal component) array. (rayleighs) (size: $NN_i$)

- $S1_i^o$ - Variance array for $J1^o$. (rayleighs$^2$)

- $S2_i^o$ - Variance array for $J2^o$.

- $S3_i^o$ - Variance array for $J3^o$.

- $ntherm$ - Number of thermospheric measurements being used during the green line thermospheric/mesospheric coupling.

- $idec1_h$ - Index specifying requested deconvolution scenario for J1. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $idec2_h$ - Index specifying requested deconvolution scenario for J2 and J3. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

## HYPOTHESIS OF APPLICABILITY:
None.

## CRITERIA OF APPLICABILITY:
Reduction of inversion window is requested. (*index*=1)

## TRANSFORMATION EQUATION:

(1) Use the minimum of $\sqrt{S1_i^o}$ and one percent of the maximum value as the lower limit for $J1_i^o$ that is applied on the top side of the array. Then eliminate the consecutive top side rows which fall below this lower limit. Remember that the resulting size of the arrays (i.e. $NN_i$) must be no less than four in order to apply deconvolutions later.

Sample algorithm

**Do** $i = 1, NN_i$

   **If** $\left(J1_i^o . gt.\min\left(\sqrt{S1_i^o}, 0.01 J_{max}\right)\right)$ **goto** 10

**End do**

10   **If** ($i.eq.1$) **Return**

   **If** ($i.gt.NN_i$ - 3) **then**

     $idec1_h = 0$

     $idec2_h = 0$

     **Return**

   **End if**

   **Do** $j = i, NN_i$

     $zcc_{j-i+1} = zcc_j$

     $J1_{j-i+1}^o = J1_j^o$

     $S1_{j-i+1}^o = S1_j^o$

   **End do**

   **If** ($idec2_h.gt.0$) **then**

     **Do** $j = i, NN_i$

       $J2_{j-i+1}^o = J2_j^o$

       $J3_{j-i+1}^o = J3_j^o$

       $S2_{j-i+1}^o = S2_j^o$

       $S3_{j-i+1}^o = S3_j^o$

     **End do**

   **End if**

   $NN_i = NN_i - i + 1$

(2) The number of green line thermospheric measurements, defined when ntherm>0, is reduced in size by $i$ - 1 (i.e.: *ntherm* = *ntherm* - $i$ + 1). If the resulting quantity is less than zero then it is set to zero.

### 9.1.4.3   A32143: Add layers above inversion window

#### FUNCTIONAL DESCRIPTION:

Add layers above the inversion window in order to account for high thermospheric emissions. This algorithm is applied only for the species of $O(^1S)$, $O(^1D)$, or $O^+$.

#### INPUT:

- *itermax* - Maximum number of iterations for skewed gaussian fitting. (0 - fitting is not allowed, >0 - maximum number of iterations, fitting is attempted only when layers have been added above the inversion window, <0 - its absolute value is the maximum number of iterations, fitting is attempted at all times) (from CDB)

- *index* - Index specifying how solution window size is to be changed. (0 - No change, 1 - Reduction, 2 - Add layers)

- *idec1$_h$* - Index specifying requested deconvolution scenario for J1. (0 - no deconvolutions, >0 - deconvolution scenario)

- *chi4$_h$* - Initilization value for chi-square statistic ratio for $E_h$ with respect to the skewed gaussian fit solution.

- $X_{h,k,l}$ - Skewed gaussian fit parameters. (size: 4x2 for each $h$ value)

- $J_{max}$ - Maximum apparent quantity value. (rayleighs)

- $iJ_{max}$ - Layer index of $J_{max}$.

- $NN_i$ - Dimension of arrays.

- $zcc_i$ - Tangent height (altitude) array. (size: $NN_i$)

- $J1_i^o$ - Apparent quantity array. (size: $NN_i$)

- $N_F$ - Filter number (from A11)

### OUTPUT:

- *idec1$_h$* - Index specifying requested deconvolution scenario for J1. (0 - no deconvolutions, >0 - deconvolution scenario)

- *idec2$_h$* - Index specifying requested deconvolution scenario for J2 and J3. (0 - no deconvolutions, >0 - deconvolution scenario)

- *NA* - Number of layers added above inversion window.

- $zcc_i$ - Altitude array. Includes any layer added above inversion window (work array) (size: $NN_i + NA$)

- *chi4$_h$* - Initilization value for chi-square statistic ratio for $E_h$ with respect to the skewed gaussian fit solution.

### HYPOTHESIS OF APPLICABILITY:

This algorithm is applied only for the addition of high thermospheric layers and for the species of $O(^1S)$, $O(^1D)$, or $O^+$.

### CRITERIA OF APPLICABILITY:

Addition of layers is requested (*index* = 2) and fitting using skewed gaussians is allowed ($itermax \neq 0$).

### TRANSFORMATION EQUATION:

The determination of the upper boundary for this extended region relies on a skewed gaussian based analytical model defined by a fit of the volume emission rate profile solved during the previous call to A32, or, after each filter change, by fit parameter estimates. The volume emission rate model is defined as

$$E(z, X) = \begin{cases} \sum_{l=1}^{2} X_{1,l} \, exp\left\{ -\ln(2) \left( \frac{z - X_{3,l}}{X_{2,l} + (_z - X_{3,l}) X_{4,l}} \right)^2 \right\} & \text{for } O(^1S), N_F = 2 \\ \\ X_{1,1} \, exp\left\{ -\ln(2) \left( \frac{z - X_{3,1}}{X_{2,1} + (_z - X_{3,1}) X_{4,1}} \right)^2 \right\} & \text{otherwise} \end{cases}$$

where the $X_{k,l}\,(= X_{h,k,l})$ are the fit parameters and $z$ denotes altitude.

If the altitude of maximum thermospheric apparent quantity is at the highest tangent point altitude ($zcc_1$, $i_{J_{max}} = 1$), and if the estimated altitude $X_{h,3,1}$ of maximum volume emission rate in the thermosphere is greater or close to that same tangent height, then any deconvolution solutions that would be produced should be considered unreliable. Therefore, the requested deconvolutions will not be done. The indices $idec1_h$ and $idec2_h$ are both set equal to zero and $chi4_h$ is set equal to 100 (or some other large value).

Otherwise, the upper boundary altitude of the extended window is defined as the altitude corresponding to a volume emission rate value close to 5% of the volume emission rate peak ($X_{h,1,1}$ or $X_{h,1,2}$). An upper altitude limit of 500 km is also imposed.

The extension region above the inversion window is subdivided into homogeneous layers with constant thickness $h_c$ equal to that of the first underlying layer ($h_c = zcc_1 - zcc_2$).

The final altitude array $zcc$ which includes both the new layers and the layers of the inversion window is defined using

$Z_i = zcc_i$ , for $i = 1$ to $NN_i$

$zcc_i = Z_1 + (NA + 1 - i)h_c$ , for $i = 1$ to $NA$

$zcc_{i + NA} = Z_i$ , for $i = 1$ to $NN_i$

where $NA$ is the number of layers added above the inversion window. $NA$ is determined using the 5% cutoff value (or the 500 km limit) described above and the altitude limit expression $Z_1 + NA\, h_c$ for the region above the inversion window. The final dimension of $zcc$ is $NN_i + NA$.

### 9.1.5 A3215: Calculation of line-of-sight matrix

#### FUNCTIONAL DESCRIPTION:

Determination of the line-of-sight matrix using the infinitessimal field-of-view assumption. The matrix elements are calculated using the Pythagorean theorem. The resulting matrix will be square and lower triangular when atmospheric layers have not been added above the inversion window. Otherwise, all the elements of the first few columns will be non-zero, while the remaining part of the matrix will be square and lower triangular. The matrix elements are in $\dfrac{rayleigh}{photons \bullet cm^{-3} \bullet sec^{-1}}$.

The square and lower triangular matrix result from selecting the atmospheric layers such that their altitude boundaries closely correspond to tangent height boundaries.

### INPUT:

- $NA$ - Number of layers added above inversion window.
- $NN_i$ - Number of layers for inversion window.
- $R_e$ - Reference earth radius. ( km)
- $zcc_i$ - Geodetic tangent height (and altitude) array. (km) (size: $NN_i + NA$)
- *itherm* - Index specifying if the initial and the first and/or second additional arrays are present. (0 - Neither the first nor the second additional arrays are present, 1 - the first additional vector is availble, 2 - the second additional vector is available, 3 - both additional vectors are available.) This index is intended mainly for use with green line measurements.
- *ntherm* - Number of thermospheric measurements being used during the green line thermospheric/mesospheric coupling.

### OUTPUT:

- $L_{i,j}$ - Line-of-sight matrix. (size: $NN_i$ x ($NN_i + NA$))

### HYPOTHESIS OF APPLICABILITY:

1) Local sphericity can be assumed.
2) The infinitessimal field-of-view assumption is adequate in defining the path length through each atmospheric layer (i.e., shell).
3) Spherical homogeneity can be assumed.

### CRITERIA OF APPLICABILITY:
None

### TRANSFORMATION EQUATION:

The line-of-sight matrix element $L_{i,j}$ for each infinitessimal field-of-view $i$, and through each atmospheric layer $j$, is defined, using the Pythagorean theorem, as

$$L_{i,j} = 2c\left[\sqrt{(z_{j-1} - x_i)\,(z_{j-1} + x_i + 2R_e)} - \sqrt{(z_i - x_i)\,(z_j + x_i + 2R_e)}\right]$$

where $R_e$ (*km*) is the geodetic earth radius, the $x_i$ (*km*) are the reference tangent heights for the infinitessimal fields-of-view, and $z_j$ (*km*) define layer altitude boundaries with $z_j < z_j$ - 1. The constant $c$ (=0.1) is the product of the *km* to *cm* and the

$photons \bullet cm^{-2} \bullet sec^{-1}$ to *rayleigh* conversion factors, respectively equal to $10^5$ and $10^{-6}$. The factor of 2 accounts for the line-of-sight crossing the atmospheric layer twice. The

matrix elements are in $\dfrac{rayleigh}{photons \bullet cm^{-3} \bullet sec^{-1}}$.

For the inversion window, the altitude boundaries of the atmospheric layers are approximated by the midpoint point altitudes between successive geodetic tangent heights

(i.e.: $zcc_i$). Above the inversion window, the layers are of thickness equal to that of the first inversion window layer.

The one exception is at the green line mesosphere/thermosphere interface. In this situation, the mesosphere/ thermosphere altitude boundary is defined as the lower boundary of the thermospheric measurements

(i.e.: for $ntherm > 1, z = 1.5zcc_{ntherm + NA} - 0.5zcc_{ntherm + NA - 1}$).

Sample algorithm

> **For** $i = 1$ to $NN_i$ **do**
> $$L_{i, i + NA} = 0.2 \cdot \sqrt{(z_{i + NA} - x_i)(z_{i + NA} + x_i + 2R_e)}$$
> $$b = L_{i, i + NA}$$
> **If** $(NA > 0$ and $i = 1)$ or $(i > 1)$ **then**
> > **For** $j = i + NA$ -1 to 1 by -1 **do**
> > $$c = 0.2 \cdot \sqrt{(z_j - x_i)(z_j + x_i + 2R_e)}$$
> > $$Li, j = c - b$$
> > $$b = c$$
> > **End do**
> **End if**
> **End do**

The atmospheric layer boundary $z_j$ is defined by

> **If** $(j.eq.1)$ **then**
> > $z_j = 1.5zcc_1 - 0.5zcc_2$
> **Else if** $(j.eq.NN_i + NA + 1)$ **then**
> > $z_j = 1.5zcc_{NNi+NA} - 0.5zcc_{NNi + NA - 1}$
> **Else if** $(itherm.ne.0.and.ntherm + NA + 1.eq.j.and.j.gt.2)$ **then**
> > $z_j = 1.5zcc_j - 1 - 0.5zcc_j - 2$
> **Else if** $(itherm.ne.0.and.ntherm + NA + 1.eq.j.and.j.eq.2)$ **then**
> > $z_j = (zcc_1 + 1.5zcc_2 - 0.5zcc_3) / 2$
> **Else**
> > $z_j = (zcc_j + zcc_j - 1) / 2$
> **End if**

The tangent height $x_i$ of the representative infinitesimal FOV is given by

> **If** $(itherm.ne.0.and.ntherm + 1.eq.i.and.i.lt.NN_i)$ **then**
> > $x_i = z_{i + NA + 1} + 0.5684(z_{i + NA + 1} - z_{i + NA + 2})$
> **Else if** $(itherm.ne.0.and.ntherm + 1.eq.i.and.i.eq.NN_i)$ **then**
> > $x_i = zcc_{i + NA}$
> **Else if** $(i.gt.1)$ **then**
> > $x_i = 0.5684z_{i + NA} + (1.0000 - 0.5684) z_{i + NA + 1}$
> **Else if** $(i.eq.1)$ **then**

$$x_i = 0.5684(1.5zcc_{1 + NA} - 0.5zcc_{2 + NA}) + (1.0000 - 0.5684) z_{i+NA+1}$$
**End if**

and $L_{i,j} = 0.0$ for $j - NA > i$.

The constant 0.5684 provides an integration line-of-sight that is slightly above the bin field-of-view midpoint (which corresponds to zcci+NA). This value is chosen in an attempt to minimize the error resulting from the infinitesimal field-of-view assumption.

Figure A32.1 describes the matrix element $L_{i \cdot j}$ as the sum of the line-of-sight segments AB and CD.

## 9.2 A322: Volume emission rate deconvolution

FUNCTIONAL DESCRIPTION:

**1.** Objective

Given the apparent quantity array $J1_h$, the volume emission rate profile $E_h$ is determined prior to solving for the visiblity and phase profiles. The volume emission rate profile is determined through Twomey deconvolution. If requested, the volume emission rate profile can afterwards be fitted by skewed gaussians. The fit parameters defining the skewed gaussians are determined iteratively. These fit parameters may be required later for the deconvolution of the next set of measurements. A standard deviation estimate array is provided for the volume emission rate solution. The ratio of the chi-square statistic for the apparent quantity array $J1_h$ to the 95% confidence interval chi-square statistic is provided as additional quality parameter.

**2.** General solution method

The general Twomey method equation applied for deriving the volume emission rate profile $E_h$ is

$$\left(L^T S_{J1_h}^{-1} L + \gamma_E H_E\right) E_h = L^T S_{J1_h}^{-1} J1_h \tag{EQ 9.24}$$

where $S_{J1_h}$ is the provided diagonal covariance matrix estimate, $\gamma_E$ is a given weighting factor, and $H$ is the constraint matrix. The equation can be solved using a Nag routine that accounts for a positive definite and symmetric matrix. Once the matrix equation is solved, a lower bound of zero is applied to the volume emission rates.

One possible choice for the constraint matrices is the square, i.e. $H = K^T K$, of the first order finite difference matrix operator $K$, where, denoting the length of the array $E_h$ as $N$, the $NxN$ matrix is

$$K = \begin{bmatrix} 0 & 0 & . & . & . & & \\ \dfrac{1}{E_{h,1}} & -\dfrac{1}{E_{h,2}} & 0 & . & . & . & . & & 0 \\ 0 & \dfrac{1}{E_{h,2}} & -\dfrac{1}{\hat{E}_{h,3}} & 0 & . & . & & . & & 0 \\ \vdots & & & & & & & \\ . & & & & . & 0 & \dfrac{1}{\hat{E}_{h,N-1}} & -\dfrac{1}{\hat{E}_{h,N}} \end{bmatrix} \tag{EQ 9.25}$$

The array $\hat{E}_h$ may be based on the solution of the previous deconvolution period, or, if available, a modelled version of the latter, or a combination of the above two possibili-

ties. This modelled version will have been defined by fitting skewed gaussians to this previous deconvolution solution.

The above constraint matrix could be used when the deconvolution window has been extended, or when it is desired that the weight of the constraint be approximately proportional to the ratio of the emission rate random error variance over the square of the volume emission rate. The random error information is contained in the first term of the deconvolution equation, while an estimate of the required volume emission rate is imbedded in the finite difference operator.

### 3. Quality parameters

The standard deviation estimate array $\sigma_{E_h}$ for the volume emission rate profile $E_h$ is calculated using

$$\sigma_{E_{hi}}^2 = \left( L^T S_{J1_h}^{-1} L \right)_{i,i}^{-1} \tag{EQ 9.26}$$

The chi-square statistic ratio calculated for $J1_h$ is defined as

$$chi1_h = \frac{\left\| S_{J1_h}^{-1/2} (LE_h - J1_h) \right\|^2}{NN_i + 2\sqrt{2NN_i}} \tag{EQ 9.27}$$

### 4. Skewed gaussian fitting

The profile required in defining the matrix $K$ shown above should vary smoothly with altitude and should exhibit, whenever possible, the principle features of the volume emission rate profile. In addition, for high thermospheric emissions, $E_h$ must extend above the initial inversion window. For this purpose, skewed gaussians were selected to model the constraint volume emission rate profile. The volume emission rate model is defined as

$$E(z, X) = \begin{cases} \displaystyle\sum_{l=1}^{2} X_{1,l} \exp\left\{ -\ln(2) \left( \frac{z - X_{3,l}}{X_{2,l} + (z - X_{3,l}) X_{4,l}} \right)^2 \right\} & \text{for O($^1$S)} \\[3ex] X_{1,l} \exp\left\{ -\ln(2) \left( \frac{z - X_{3,l}}{X_{2,l} + (z - X_{3,l}) X_{4,1}} \right)^2 \right\} & \text{otherwise} \end{cases} \tag{EQ 9.28}$$

where $z$ denotes altitude. (Thuillier, 1973).

The value of the parameters $X_{k,l}$ are determined, through iteration, by minimizing

$$\psi^2 = \frac{\sum_i \frac{1}{\max(E_{hi}, 1)} \frac{(E_{hi} - \bar{E}(z_i, X))^2}{\sigma_{E_{hi}}^2}}{\sum_j \frac{1}{\max(E_{hj}, 1)}}$$

(EQ 9.29)

where $\bar{E}(z_i, X$ is a model average for atmospheric layer $i$, and $E_h$ is the deconvolution volume emission rate profile. For a volume emission rate profile of length $NN_i$, a convergence criterion is $\psi^2 \leq NN_i + 2\sqrt{2NN_i}$. A limit on the number of iterations is imposed. For the window extention case, if an acceptable fit cannot be determined, deconvolutions cannot be done.

The initial fit parameter values are typically those defined during the previous deconvolution. After the first Twomey deconvolution following a filter change, other than during subsequent switching of the OH filters, the altitude $X_3$ and value $X_1$ of the maximum emission rate can be defined using the solution profile $E_h$, and the half-width $X_2$ initialized as the average of the upper and lower half-widths of $E_h$ (if both are available). The half-width is defined as the magnitude of the altitude difference between the positions of $E_{max}$ and $E_{half}$ where $E_{half} \approx \frac{1}{2} E_{max}$.

### INPUT:

- $\gamma_E$ - Weighting factor. (from CDB)

- $MAT1$ - Index specifying constraint matrix for $H_E$. (from CDB)

- $idec1_h$ - Index specifying requested deconvolution scenario for J1. (0 - no deconvolution, >0 - deconvolution scenario)

- $itermax$ - Maximum number of iterations for skewed gaussian fitting. (0 - fitting is not allowed, >0 - maximum number of iterations, fitting is attempted only when layers have been added above the inversion window, <0 - its absolute value is the maximum number of iterations, fitting is attempted at all times) (from CDB)

- $chi4_h$ - Chi-square statistic ratio for $E_h$ with respect to the skewed gaussian fit solution. (for previous fit solution if available)

### HYPOTHESIS OF APPLICABILITY:
None.

### CRITERIA OF APPLICABILITY:
The deconvolution for volume emission rates is requested. The apparent quantity array $J1_h$ has a length greater or equal to four. ($idec1_h > 0$)

**TRANSFORMATION EQUATIONS:**

(1) If skewed gaussian fitting is allowed ($itermax \neq 0$) and the first guess values for the fit parameters are available ($chi4_h \leq 1$), then if $MAT1$ is not equal to 7, 9, or 10, $MAT=10$ should be used instead of $MAT1$, and if $\gamma_E$ is less than 0.1, $\gamma = 0.1$ should be used in place of $\gamma_E$. This ensures that the region above the inversion window will be taken into account in the deconvolution.

(2) Call A3221 - If requested, define the reference volume emission rate profiles. (see step (1))

(3) Call A3222 - If requested, define the constraint matrix $H_E$. (see step (1))

Input: $idec1_h$, $MAT1$, $NA$, $NN_i$, $x$, $\tilde{x}$, $i_{add}$, $\gamma_E$, $L$, $S1^o$
Output: $H_E$

(4) Call A3223 - Solve for the volume emission rate profile using the Twomey matrix equation.

Input: $idec1_h$, $NA$, $NN_i$, $L$, $J1^o$, $S1^o$, $\gamma_E$, $H_E$
Output: $idec1_h$, $E^o$

(5) Call A3224 - Define the final volume emission rate profile.

(6) Call A3225 - Derive the chi-square statistic ratio for $J1_h$.

Input: $idec1_h$, $L$, $J1^o$, $S1^o$, $NA$, $NN_i$, $Eo$
Output: $chi1_h$

(7) Call A3226 - Derive the standard deviation estimate array.

(8) Call A3227 - If requested, apply skewed gaussian fitting to the volume emission rate profile by minimizing

$$X^2 = \sum_i \frac{(E_{hi} - \bar{E}(z_i, X))^2}{\sigma_{E_{hi}}^2}$$

(EQ 9.30)

### 9.2.1   A3221: Define reference profiles for $E_h$.

**FUNCTIONAL DESCRIPTION:**
Define either one or two reference profiles for $E_h$. The reference profiles are defined using a selection index. At the moment only four choices are available. If requested, these reference profiles will be used when defining the Twomey deconvolution equations.

**INPUT:**

- $iref1$ - Index specifying selection of reference profiles for $E_h$. (from CDB)

- $\gamma_E$ - Weighting factor. (from CDB)

- $NP_h$ - Dimension of previous solution arrays.

- $z_{p_{hi}}$ - Altitude array for previous solution arrays. (km) (size: $NP_h$) If $NP_h = 0$, $z_p$ is not available.

- $E_{p_{hi}}$ - Previous volume emission rate solution array. (photons cm^-3 s^-1) (size: $NP_h$). If $NP_h = 0$, $E_p$ is not available.

- $NP_{F_h}$ - Filter number for previous deconvolution.

- $N_F$ - Current filter number.

- $itermax$ - Maximum number of iterations for skewed gaussian fitting. (0 - fitting is not allowed, >0 - maximum number of iterations, fitting is attempted only when layers have been added above the inversion window, <0 - its absolute value is the maximum number of iterations, fitting is attempted at all times) (from CDB)

- $chi4_h$ - Chi-square statistic ratio for $E_h$ with respect to the skewed gaussian fit solution. (for previous volume emission rate fit solution)

- $X_{k,l}$ - Initial skewed gaussian fit parameters. (size: 4x2)

- $idec1_h$ - Index specifying requested deconvolution scenario for J1. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $NN_i$ - Number of layers for inversion window.

- $NA$ - Number of layers above inversion window.

- $zcc_i$ - Altitude array. (km) (size: $NN_i + NA$)

- $J1_i^o$ - Apparent emission rate array. (rayleighs) (size: $NN_i$)

- $L_{i,j}$ - Line-of-sight matrix. (10's of km) (size: $NN_i$ x ($NN_i + NA$))

**OUTPUT:**

- $idec1_h$ - Index specifying requested deconvolution scenario for J1. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $x_i$ - Reference profile for possible use in defining constraint matrix $H_E$. (size: $NN_i + NA$)

- $\tilde{x}_i$ - Additional reference profile (size: $NN_i + NA$). Only available when indicated (i.e. when $i_{add} = 1$)

- $i_{add}$ - Indicates if $\tilde{x}_i$ is available. (=0 - not available, =1 - available)

**HYPOTHESIS OF APPLICABILITY:**
None.

## CRITERIA OF APPLICABILITY:

The deconvolution of $J1^o$ is requested ($idec1_h > 0$), and reference profile(s) are required ($\gamma_E > 0$).

## TRANSFORMATION EQUATION:

The reference profile $x$ is specified using the selection index $iref1$. At the moment only four choices are available for these profiles. The default value for $iref1$ is set equal to 1 if the initial $iref1$ is out of the range for the available choices.

If $iref1 < 3$, skewed gaussian fitting is allowed ($itermax \neq 0$), and the first guess fit parameters are available ($chi4_h \leq 1$), then the third choice is used instead of that specified by $iref1$.

An additional reference profile, denoted by $\tilde{x}$, is defined using the previous volume emission rate solution when the latter available. Otherwise (i.e. $NP_{F_h} \neq N_F$ except when OH filters are being switched), $\tilde{x}$ is set equal to $x$. When $E_{p_h}$ is available, $i_{add}$ is set equal to 1 and  is defined using choice four specified below.

List of available choices for $x$

(1) Set each element in $x$ as equal to 1.0.

(2) If $NA = 0$, the array elements xi are set equal to $\dfrac{J1^o_i}{L_{i,i}}$ with an imposed lower limit of 1.0. Otherwise, $idec1_h$ is set equal to zero.

(3) Define $x$ using only the skewed gaussian model described below if the fit parameters are available. If they are not, use the second choice when $NA = 0$, and, when $NA > 0$, specify that no deconvolutions will be possible (set $idec1_h = 0$) and then exit this algorithm.

The elements $x_i$ are set equal to the layer average $\bar{E}(zcc_i, X)$ where the latter can be defined using Simpson's rule, e.g.

$$\bar{E}(zcc_i, X) = \frac{1}{6}[E(0.5(zcc_{i-1} + zcc_i), X) + 4E(zcc_i, X) + E(0.5(zcc_{i+1} + zcc_i), X)]$$

The modelled volume emission rate function

$$E(z, X) = \begin{cases} \displaystyle\sum_{l=1}^{2} X_{1,l} \exp\left\{-\ln(2)\left(\frac{z - X_{3,l}}{X_{2,l} + (_z - X_{3,l})}\right)^2\right\} & \text{for } O(^1S), N_F = 2 \\[20pt] X_{1,l} \exp\left\{-\ln(2)\left(\frac{z - X_{3,l}}{X_{2,l} + (_z - X_{3,l})}\right)^2\right\} & \text{otherwise} \end{cases}$$

with $X_{k,l}$ denoting the fit parameters. For $O(^1S)$, if any one of the two maximum volume emission rate values $X_{1,1}$ and $X_{1,2}$ are zero, the corresponding skewed gaussian is not used in the fit. If the skewed gaussian exponent is excessively large or if its denominator is less or equal to zero, the corresponding term in the summation for $E(z,X)$ is set equal to 0.5. A lower limit of 1.0 is applied for $E(z,X)$.

(4) If the filter has just been changed in order to observe a different species ($NP_{F_h} \neq N_F$ except when OH filters are being switched), do choice (2) for layers of the inversion window (i.e. $i > NA$). Otherwise, for layers of the inversion window ($i > NA$), set the $x_i$ for $i \leq NA$ equal to the elements resulting from the linear interpolation, in altitude, of $E_{p_h}$.

For layers above the inversion window, use the skewed gaussian fit model of choice (3). In addition, the reference profiles above the inversion window are matched to that of the inversion window by multiplying $x_i$ for $i \leq NA$ by a factor where

$$\alpha = \frac{x_{NA+1}}{E(zcc_{NA+1}, X)}$$

### 9.2.2 A3222: Selection of constraint matrix

#### FUNCTIONAL DESCRIPTION:
A constraint matrix is selected by means of an index. There are currently eleven possible choices.

#### INPUT:
- $idec$ - Index specifying deconvolution scenario. (either $idec1_h$ or $idec2_h$) (0 - no deconvolution, >0 - deconvolution scenario)
- $MAT$ - Index specifying selected constraint matrix. (either $MAT1$ (or $MAT$), $MAT2$, $MAT3$, $MAT22$, $MAT33$)
- $NA$ - Number of layers added above inversion window.
- $NN_i$ - Number of inversion window layers.
- $x_i$ - Reference profile (size: $NN_i + NA$)
- $\tilde{x}_i$ - Additional reference profile (size: $NN_i + NA$). Only available when indicated (i.e. when $i_{add} = 1$)

- $i_{add}$ - Indicates if  is available. (=0 - not available, =1 - available)

- $\gamma$ - Weighting factor. (either $\gamma_E$, $\gamma_2$, $\gamma_3$, $\gamma_{22}$ or $\gamma_{33}$)

- $L_{i,j}$ - Transformation matrix. (size: $NN_i$ x $(NN_i+NA)$) (either $L_{i,j}$ or $\tilde{L}_{i,j}$)

- $S_i^o$ - Variance array. (size: $NN_i$) (either $S1^o$, $S2^o$, or $S3^o$)

### OUTPUT:

- $C_{i,j}$ - One of the following constraint matrices: $H_E$, $H_{V^c}$, $H_{V'}$, $H_{A^c}$, or $H_{A'}$. (size: $(NN_i+NA)$ x $(NN_i+NA)$)

### HYPOTHESIS OF APPLICABILITY:
None.

### CRITERIA OF APPLICABILITY:
Deconvolution is requested (idec>0) and the constraint matrix is required (>0).

### TRANSFORMATION EQUATION:

The constraint matrix is selected from the following list using the value of *MAT*. For cases where a matrix denoted by $K$ is presented, the final constraint matrix is calculated using

$$C = K^T K$$

The calculation of the matrix $C$ should be done such that computing time is minimized (e.g. account for sparsity of $K$ and symmetry of $C$). The $K$ matrices are finite difference representations of low order differential operators. For example, the second difference matrix of type 3 ($MAT = 5$) is representative of the operator

$$x \frac{\partial^2 \frac{1}{x}}{\partial z^2}$$

where $z$ denotes altitude.

For any $x_i$ value less in magnitude than about 0.001, this limiting value is instead applied in defining the corresponding elements in the following matrices. The above should also be applied to $\tilde{x}$ when the latter is available.

The second difference matrix of type 4 and the first difference matrix of type 4 can only be used when the additional array $\tilde{x}$ has been made available (i.e. $i_{add} = 1$).

If $i_{add} = 0$ and $MAT = 6$ or 10 then select matrices 3 or 7 respectively.

All $K$ and $C$ matrices are $(NN_i+NA)$ x $(NN_i+NA)$. For convenience let $M = NN_i + NA$.

$$
K = \begin{bmatrix}
0 & 0 & . & & . & . \\
x_2^{-1} & -2x_2^{-1} & x_2^{-1} & 0 & . & . & . & & . & & 0 \\
0 & x_3^{-1} & -2x_3^{-1} & x_3^{-1} & 0 & . & . & & . & & 0 \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & . & . & . & 0 & x_{M-1}^{-1} & -2x_{M-1}^{-1} & x_{M-1}^{-1} \\
0 & . & & . & & . & . & . & 0 & 0 & 0
\end{bmatrix}
$$

(5) Second difference matrix

$$
K = \begin{bmatrix}
0 & 0 & . & . & . \\
\dfrac{x_2}{x_1} & -2 & \dfrac{x_2}{x_3} & 0 & . & . & . & & . & & 0 \\
0 & \dfrac{x_3}{x_2} & -2 & \dfrac{x_3}{x_4} & 0 & . & . & & . & & 0 \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & . & . & 0 & \dfrac{x_{M-1}}{x_{M-2}} & -2 & \dfrac{x_{M-1}}{x_M} \\
0 & . & & . & . & . & & 0 & 0 & 0
\end{bmatrix}
$$

(6) Second difference matrix

$$
K = \begin{bmatrix}
0 & 0 & . & . & . \\
\dfrac{x_2}{x_1\sqrt{\tilde{x}_2}} & -\dfrac{2}{\sqrt{\tilde{x}_2}} & \dfrac{x_2}{x_3\sqrt{\tilde{x}_2}} & 0 & . & . & . & . & 0 \\
0 & \dfrac{x_3}{x_2\sqrt{\tilde{x}_3}} & -\dfrac{2}{\sqrt{\tilde{x}_3}} & \dfrac{x_3}{x_4\sqrt{\tilde{x}_3}} & 0 & . & . & . & 0 \\
. & & & & & & & & \\
. & & & & & & & & \\
. & & & & . & . & 0 & \dfrac{x_{M-1}}{x_{M-2}\sqrt{\tilde{x}_{M-1}}} & -\dfrac{2}{\sqrt{\tilde{x}_{M-1}}} & \dfrac{x_{M-1}}{x_M\sqrt{\tilde{x}_{M-1}}} \\
0 & . & & . & & . & . & . & 0 & 0 & 0
\end{bmatrix}
$$

(7) First difference matrix

$$K = \begin{bmatrix} 0 & 0 & \cdot & \cdot & & & \\ \dfrac{x_2}{x_1\sqrt{\tilde{x}_2}} & -\dfrac{1}{\sqrt{\tilde{x}_2}} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \dfrac{x_3}{x_2\sqrt{\tilde{x}_3}} & -\dfrac{1}{\sqrt{\tilde{x}_3}} & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & & & \\ \cdot & & & & & & \\ \cdot & & & \cdot & \cdot & 0 & \dfrac{x_M}{x_{M-1}\sqrt{\tilde{x}_M}} & -\dfrac{1}{\sqrt{\tilde{x}_M}} \end{bmatrix}$$

(11) Inverse of covariance matrix for measurements

$$C = L^T S_J^{-1} L$$

where $S_J$ is diagonal with $S_{J_{i,i}} = S_i^o$.

The calculation of matrix products should be done such that computing time is minimized (e.g. account for the sparsity of $L$ and the symmetry of $L^T S_J^{-1} L$).

### 9.2.3 A3223: Twomey deconvolution

**FUNCTIONAL DESCRIPTION:**
A solution profile is determined using a least-squares linear constrained deconvolution method based on Twomey (1977).

**INPUT:**
- *idec* - Index specifying deconvolution scenario. (either $idec1_h$ or $idec2_h$)
- *NA* - Number of layers added above inversion window.
- $NN_i$ - Dimension of measurement array and number of layers for inversion window layers.
- $L_{i,j}$ - Transformation matrix. (either $L_{i,j}$ or $\tilde{L}_{i,j}$) (size: $NN_i$ x ($NN_i$+$NA$))
- $J_i$ - Apparent quantity array. (either $J1^o$, $J2^o$, $J2'$, $J3^o$ or $J3'$) (*rayleighs*) (size: $NN_i$) (rayleighs)
- $S_i^o$ - Variance array. (either $S1^o$, $S2^o$, or $S3^o$) (size: $NN_i$)
- $\gamma$ - Weighting factor for constraint matrix $H$. (either $\gamma_E$, $\gamma_2$, $\gamma_3$, $\gamma_{22}$, or $\gamma_{33}$ )
- $H$ - Constraint matrix. (size: ($NN_i$+$NA$) x ($NN_i$+$NA$)) (either $H_E$, $H_{V^o}$, $H_{A^o}$, $H_{V'}$, or $H_{A'}$) (not available nor necessary if $\gamma \le 0$)

<u>OUTPUT:</u>

- *idec* - Index specifying deconvolution scenario. Indicates if the deconvolution was successful (>0) or not (=0). (either $idec1_h$ or $idec2_h$)

- *x* - Solution of deconvolution process. (either $E^o$, $V^c$, $A^c$, $V^s$, or $A^s$) (size: $NN_i + NA$)

<u>HYPOTHESIS OF APPLICABILITY:</u>

None.

<u>CRITERIA OF APPLICABILITY:</u>

Twomey deconvolution is requested ($idec > 0$).

<u>TRANSFORMATION EQUATION:</u>

(1) Solve for x in

$$(L^T S_J^{-1} L + \gamma H) x = L^T S_J^{-1} J$$

where $S_J$ is diagonal with $S_{J_{i,i}} = S_i^o$. The equation can be solved using a Nag routine that accounts for a positive definite and symmetric matrix for a system of the type $Bx=y$ where $B$ is the matrix.

The calculation of matrix products or additions should be done such that computing time is minimized (e.g. account for the sparsity of $L$ and the symmetry of $L^T S_J^{-1} L$ and $H$).

(2) If an acceptable solution could not be determined by the Nag routine, the index idec is set equal to a non-positive number. ($idec \le 0$)

## 9.2.4   A3224: Set final volume emission rate profile

<u>FUNCTIONAL DESCRIPTION:</u>

Eliminate volume emission rate solution elements for layers above the inversion window and any negative solution values.

<u>INPUT:</u>

- $idec1_h$ - Index specifying if deconvolution scenario for J1 was successful. (0 - failed or no deconvolution, >0 - deconvolution scenario)
- *NA* - Number of layers added above inversion window.
- $NN_i$ - Dimension of measurement array and number of layers for inversion window layers.
- $E_i^o$ - Volume emission rate profile obtained from deconvolution of J1. (size: $NN_i + NA$)
- $zcc_i$ - Altitude array. (km) (size: $NN_i + NA$)

**OUTPUT:**

- $E_{hi}$ - Final volume emission rate profile obtained for the inversion window. (size: $NN_i$)

- $zcc_i$ - Altitude array. (km) (size: $NN_i$)

**HYPOTHESIS OF APPLICABILITY:**

None.

**CRITERIA OF APPLICABILITY:**

Deconvolution for $E_h$ was successful. $(idec1_h > 0)$

**TRANSFORMATION EQUATION:**

All negative elements in the array $E^o$ are set equal to zero.

The volume emission rate profile is defined for the inversion window by discarding the solution values for the layers above the window, i.e.

$$E_{h_i} = E^o_{i+NA}$$

and

$$zcc_i = zcc_{i+NA}$$

for $i = 1$ to $NN_i$

### 9.2.5  A3225: Chi-square statistic ratio calculation

**FUNCTIONAL DESCRIPTION:**

The ratio of the solution related chi-square statistic to the chi-square 95th percentile is determined for an apparent quantity array.

**INPUT:**

- $idec$ - Index specifying requested deconvolution scenario. (0 - failed or no deconvolution, >0 - deconvolution scenario) (either $idec1_h$ or $idec2_h$)

- $L_{i,j}$ - Transformation. (either $L_{i,j}$ or $\tilde{L}_{i,j}$) (size: $NN_i$ x $(NN_i+NA)$)

- $J_i$ - Apparent quantity array. (either $J1^o$, $J2^o$, or $J2^o$, or $J3^o$ or $J3^o$) (size: $NN_i$)

- $S1^o_i$ - Variance array for $J$. (either $S1^o$, $S2^o$, or $S3^o$) (size: $NN_i$)

- $NA$ - Number of layers above inversion window.

- $NN_i$ - Number of layers for inversion window.

- $x_i$ - Deconvolution solution profile. (either $E^o$, $V^c$, $A^c$, $V^s$, or $A^s$) (size: $NN_i+NA$)

OUTPUT:

- $chi$ - Chi-square statistic ratio. (either $chi1_h$, $chi2_h$, or $chi3_h$)

HYPOTHESIS OF APPLICABILITY:

None.

CRITERIA OF APPLICABILITY:

Applied only if deconvolution was successful. ($idec > 0$)

TRANSFORMATION EQUATION:

The chi-square statistic ratio is given by

$$chi = \frac{\left\| S^{-1/2}(Lx - J)\right\|^2}{NN_i + 2\sqrt{2NN_i}}$$

where the diagonal covariance matrix $S$ has its diagonal equal to $S^o$.

### 9.2.6   A3226: Derive standard deviation estimate array for $E_h$

FUNCTIONAL DESCRIPTION:

The standard deviation of each volume emission rate array element is determined, based on direct inversion, given the variance array associated with the apparent quantity vector.

INPUT:

- $idec1_h$ - Index specifying deconvolution scenario. (0 - failed or no deconvolution, >0 - deconvolution scenario)

- $L_{i,j}$ - Line-of-sight matrix (size: $NN_i$ x ($NN_i$+$NA$))

- $S1_i^o$ - Variance array for the radiance profile $J1_h$. (size: $NN_i$)

- $NN_i$ - Dimension of arrays.

- $NA$ - Number of layers added above inversion window.

OUTPUT:

- $\sigma_{E_{hi}}$ - Standard deviation profile for volume emission rates. (size: $NN_i$)

HYPOTHESIS OF APPLICABILITY:

None.

CRITERIA OF APPLICABILITY:

The deconvolution was successful ($idec1_h > 0$).

TRANSFORMATION EQUATION:

The derivation of standard deviations is based on the equation

$$\sigma_{E_{hi}} = \sqrt{\left(L^T \bullet S_{J1_h}^{-1} \bullet L\right)^{-1}_{i+NA,\, i+NA}}$$

where $i = 1$ to $NN_i$, and $S_{J1_h}$ is a diagonal covariance matrix, with the diagonal equal to the variance array $S1^o$.

### 9.2.7 A3227: Skewed gaussian fitting

FUNCTIONAL DESCRIPTION:

One or two skewed gaussians are fitted to the volume emission rate profile of the inversion window. The fitting involves finding acceptable values for either four or eight fit parameters. This search is done using an iterative routine selected from the Nag library.

The fit parameters are the maximum volume emission rate $X_1$, the altitude $X_3$ of maximum volume emission rate, the reference half-width $X_2$, and the skewness factor $X_4$ for each of either one or two (for $O(^1S)$) emission regions.

INPUT:

- $NP_{F_h}$ - Filter number for previous deconvolution. ($N_F$ of previous call to A32)

- $N_F$ - Current filter number.

- *itermax* - Maximum number of iterations for skewed gaussian fitting. (0 - fitting is not allowed, >0 - maximum number of iterations, fitting is attempted only when layers have been added above the inversion window, <0 - its absolute value is the maximum number of iterations, fitting is attempted at all times) (from CDB)

- $chi4_h$ - Chi-square statistic ratio for $E_h$ with respect to the skewed gaussian fit solution. (for previous fit solution if available)

- $X_{h,k,l}$ - Initial skewed gaussian fit parameters. (size: 4x2 for each $h$ value)

- $idec1_h$ - Index specifying if deconvolution scenario for J1 was successful. (0 - failed or no deconvolutions, >0 - deconvolution scenario)

- $NN_i$ - Dimension of arrays.

- $E_{hi}$ - Volume emission rate profile. ($photons \bullet cm^{-3} \bullet s^{-1}$) (size: $NN_i$)

- $\sigma_{E_{hi}}$ - Volume emission rate standard deviation estimate array.

- $zcc_i$ - Altitude array. (km) (size: $NN_i$)

OUTPUT:

- $chi4_h$ - Chi-square statistic ratio for $E_h$ with respect to the skewed gaussian fit solution. (for current fit solution)

- $iconv_h$ - Number of iterations for convergence of skewed gaussian fitting. (for current fit solution)
- $X_{h,k,l}$ - Final skewed gaussian fit parameters. (size: 4x2 for each $h$)

## HYPOTHESIS OF APPLICABILITY:
None.

## CRITERIA OF APPLICABILITY:
The deconvolution was successful ($idec1_h > 0$), fitting is allowed ($itermax \neq 0$), and the first guess values for the fit parameters are available and acceptable ($chi4_h \leq 1$).

## TRANSFORMATION EQUATION:

(1) Re-define first guess fit parameters when requested:

The initial fit parameter values are typically those defined during the previous deconvolution, or those initialized in A321. Immediately after the first Twomey deconvolution following a filter change (i.e. $NP_{F_h} \neq N_F$), other than during subsequent switching of the OH filters $N_F = 5$, the altitude $X_3$ and value $X_1$ of the maximum volume emission rate are re-defined using the solution profile $E_h$, and the half-width $X_2$ is re-initialized as the average of the upper and lower half-widths of $E_h$ (if both are available). The half-width is defined as the magnitude of the altitude difference between the positions of $E_{max}$ and $E_{half}$ where $E_{half} \approx \frac{1}{2} E_{max}$.

In addition, whenever $1/2 E_{hi} > X_{h,1,l}$ or $2 E_{hi} > X_{h,1,l}$, where zcci is closest to $X_{h,3,l}$ (other similar inequality conditions could instead be used), the $X_{h,1,l}$ value is re-initialized as equal to $E_{hi}$.

For $O(^1S)$ (i.e. $N_F = 2$) where two skewed gaussians are considered, the re-initialized fit parameters are determined for the regions above and below 140 km (see A3213). Also for $O(^1S)$, if any one of the two maximum emission rate values $X_{h,1,1}$ and $X_{h,1,2}$ are zero, the corresponding skewed gaussian is not used in the fit.

(2) Fitting:

The value of the parameters $X_{h,k,l}$ are determined, through iteration, by minimizing

$$\psi^2 = \frac{\displaystyle\sum_i \frac{1}{\max(E_{hi}, 1)} \frac{(E_{hi} - \bar{E}(z_i, X))^2}{\sigma_{E_{hi}}^2}}{\displaystyle\sum_j \frac{1}{\max(E_{hj}, 1)}}$$

where $\bar{E}(zcc_i, X)$ is a model average for atmospheric layer $i$, and $E_h$ is the deconvolution solution volume emission rate profile. Only the variance array $\sigma^2_{E_h}$ is used, instead of the entire covariance matrix for $E_h$, in order to save computing time. The terms involving $(E_{hi}, 1)$ are introduced as weights.

The model average for atmospheric layer $i$ can be defined using Simpson's rule, e.g.

$$\bar{E}(zcc_i, X) = \frac{1}{6}[E(0.5(zcc_{i-1} + zcc_i), X) + 4E(zcc_i, X) + E(0.5(zcc_{i+1} + zcc_i), X)]$$

The modelled volume emission rate function $E(z,X)$ is defined as

$$\hat{E}(z, X) = \begin{cases} \sum_{l=1}^{2} X_{1,l} \exp\left\{-\ln(2)\left(\frac{z - X_{3,l}}{X_{2,l} + (z - X_{3,l})X_{4,l}}\right)^2\right\} & \text{for } O(^1S), N_F = 2 \\ X_{1,l}\exp\left\{-\ln(2)\left(\frac{z - X_{3,l}}{X_{2,l} + (z - X_{3,l})X_{4,1}}\right)^2\right\} & \text{otherwise} \end{cases}$$

with $X_{k,l} (=X_{h,k,l})$ denoting the fit parameters. For $O(^1S)$, if any one of the two maximum emission rate values $X_{h,1,1}$ and $X_{h,1,2}$ are zero, the corresponding skewed gaussian is not used in the fit. If a skewed gaussian exponent has an excessively large value, or if its denominator is less or equal to zero, the corresponding term in the summation for $E(z,X)$ can be set equal, for example, to 0.005. A lower limit of 0.01 for example can be applied for $E(z,X)$.

For a volume emission rate profile of length $NN_i$, one of the minimization convergence criteria should be

$$\psi^2 \leq NN_i + 2\sqrt{2NN_i}$$

The imposed limit on the number of iterations *iconv* is | *itermax* |, except possibly for the first attempted fitting after a filter change (e.g. measurements for $O(^1S)$ following that for OH). The maximum number of iterations during the first fitting after a filter change might have to be larger than | *itermax* |.

If convergence was not attained or if the final fit parameters do not satisfy the following inequality conditions, then the fit is considered unacceptable. In addition, if the following inequality conditions are not satisfied, the fit parameters are set equal to their initial (first guess) values of step (1).

Inequality conditions for fit parameters:

- $\left|X_{h,4,l}\right| < 5$
- $zcc_1 > X_{h,3,l} > zcc_{NNi}$

- $X_{h,3,2} > X_{h,3,1}$ when applicable
- $3E_{hi} > X_{h,1,l} >$ where $zcc_i$ is closest to $X_{h,3,l}$
- $300km > X_{h,2,l} > 2\ km$

If possible, the above inequality conditions could be imposed in the iterative method.

The initial value of the chi-square statistic ratio for $E_h$ is set equal to

$$chi4_h = \frac{\psi^2}{NN_i + 2\sqrt{2NNi}}$$

If the fit solution is not acceptable, chi4h is assigned a lower limit larger than 1.0

## 9.3 A323: Visibility (or amplitude) and wind phase deconvolution

### FUNCTIONAL DESCRIPTION:

**1.** Objective

Given the apparent quantity arrays $J2_h$ and $J3_h$, the profiles of visibility $V_h$ or amplitude $A_h$, and of wind phase $\phi_h$ are determined so that temperature and wind speed profiles can be calculated later. The most commonly requested deconvolution solutions will be those of visibilities, and wind phases. For OH (and $O_2$), the amplitude profile may be calculated instead of the visiblity profile.

Standard deviation estimate arrays are provided for the visibility, or amplitude, and phase solutions. The ratios of the chi-square statistics for the apparent quantity arrays $J2_h$ and $J3_h$ to the 95% confidence interval chi-square statistic are provided as additional quality parameters.

**2.** System reduction

Before solving for the visibility and phase profiles, numerical stability of the deconvolution equations is improved when necessary. This is done by only discarding the bottom layers with weak volume emission rates and the corresponding apparent quantities. Cut-off values of 5% of the deconvolved maximum volume emission rate and one standard deviation are applied below the peak emission rate altitude.

For cases where the deconvolution window has been extended, with the volume emission rates having been determined, the line-of-sight matrix is simplified by assuming a constant profile of visibility and phase above the initial inversion window. The layers of the extension region are combined with the top layer of the initial inversion window to form a single unit. The modified line-of-sight matrix $\bar{L}$ (see below) should then become lower triangular.

**3.** General solution method

The general Twomey method equations for the visibility profile $V_h$, and the wind phase profile $\phi_h$, are

$$\left( \tilde{L}^T S_{J2_h}^{-1} \tilde{L} + \gamma_{V^c} H_{V^c} \right) V^c = \tilde{L}^T S_{J2_h}^{-1} J2' \qquad \text{(EQ 9.31)}$$

$$\left( \tilde{L}^T S_{J3_h}^{-1} \tilde{L} + \gamma_{V^s} H_{V^s} \right) V^s = \tilde{L}^T S_{J3_h}^{-1} J3' \qquad \text{(EQ 9.32)}$$

where

$$\tilde{L}_{i,j} = L_{i,j} E_{h_j} \qquad \text{(EQ 9.33)}$$

$$\begin{bmatrix} J2_i{}' \\ J3_i{}' \end{bmatrix} = \frac{(LE_h)_i}{J1_{hi}} \begin{bmatrix} J2_{hi} \\ J3_{hi} \end{bmatrix} \qquad \text{(EQ 9.34)}$$

$$\begin{bmatrix} V_i^c \\ V_i^s \end{bmatrix} = v_{hi} \begin{bmatrix} \cos(\phi_{hi}) \\ \sin(\phi_{hi}) \end{bmatrix} \qquad \text{(EQ 9.35)}$$

where $S_{J2_h}$ and $S_{J3_h}$ are diagonal covariance matrix estimates, the $\gamma$s are weighting factors, and the $H$s are the constraint matrices. The constraint matrices are selected, from a set of pre-defined choices, using indices. The equations are solved using a Nag routine. Once $V^c$ and $V^s$ are known, the profiles of visibility $V_h$ and of wind phase $\phi_i$ are derived. Any visibility value less than 0.1 or greater than 0.99 is considered unacceptable.

**4.** Amplitude determination

For the OH (and $O_2$) species, the deconvolutions for amplitude and wind phase profiles may be requested instead of for the visibility and wind phase profiles.

The general Twomey method equations are

$$\left( L^T S_{J2_h}^{-1} L + \gamma_{A^c} H_{A^c} \right) A^c = L^T S_{J2_h}^{-1} J2_h \qquad \text{(EQ 9.36)}$$

$$\left( L^T S_{J3_h}^{-1} L + \gamma_{A^s} H_{A^s} \right) A^s = L^T S_{J3_h}^{-1} J3_h \qquad \text{(EQ 9.37)}$$

where

$$\begin{bmatrix} A_i^c \\ A_i^s \end{bmatrix} = 2A_{hi} \begin{bmatrix} \cos(\phi_{hi}) \\ \sin(\phi_{hi}) \end{bmatrix} \qquad \text{(EQ 9.38)}$$

The same algorithms used for the general solution method are employed in the amplitude determination.

**5.** Quality parameters

Once the standard deviation estimate arrays have been calculated for $E^c$ and $E^s$ (see FUNCTIONAL DESCRIPTION of A32), standard deviation estimate arrays can be determined for $V_h$ and $\phi_h$. A first step is the derivation of $\sigma_{V^c}^2$ and $\sigma_{V^s}^2$. The latter is based on the variance expression of the product of two uncorrelated variates,

$$\sigma_{ab}^2 = \sigma_a^2 \sigma_b^2 + \sigma_a^2 \mu_b^2 + \sigma_b^2 \mu_a^2 \qquad \text{(EQ 9.39)}$$

where $a$ and $b$ are the uncorrelated variates, and $\mu_a$ and $\mu_b$ are their respective means. The resulting expressions are

$$\sigma_{V_i^c}^2 = \frac{\left[\sigma_{E_i^c}^2 - (V_i^c)^2 \sigma_{E_{hi}}^2\right]}{(E_{hi})^2 + \sigma_{E_{hi}}^2} \qquad \text{(EQ 9.40)}$$

$$\sigma_{V_i^s}^2 = \frac{\left[\sigma_{E_i^s}^2 - (V_i^s)^2 \sigma_{E_{hi}}^2\right]}{(E_{hi})^2 + \sigma_{E_{hi}}^2} \qquad \text{(EQ 9.41)}$$

A lower limit of $\sigma_{E_{hi}}^2$ should be applied to the terms denoted by the square brackets.

Then, the final standard deviation estimates for each layer are determined, to first order, using

$$\sigma_{\phi_{hi}} = \frac{\sqrt{\sigma_{V_i^s}^2 (V_i^c)^2 + \sigma_{V_i^c}^2 (V_i^s)^2}}{(V_j^c)^2 + (V_j^s)^2} \qquad \text{(EQ 9.42)}$$

$$\sigma_{V_{hi}} = \sqrt{\frac{\sigma_{V_i^c}^2 (V_i^c)^2 + \sigma_{V_i^s}^2 (V_i^s)^2}{(V_j^c)^2 + (V_j^s)^2}} \qquad \text{(EQ 9.43)}$$

$$\sigma_{A_{hi}} = \frac{1}{2} \sqrt{\frac{\sigma_{A_i^c}^2 (A_i^c)^2 + \sigma_{A_i^s}^2 (A_i^s)^2}{(A_j^c)^2 + (A_j^s)^2}} \qquad \text{(EQ 9.44)}$$

The chi-square statistic ratios for $J2_h$ and $J3_h$ are defined as

$$chi2_h = \frac{\left\| S_{J2}^{-1/2} (LE^c - J2_h) \right\|^2}{NN_i + 2\sqrt{2NN_i}} \qquad \text{(EQ 9.45)}$$

$$chi3_h = \frac{\left\| S_{J3}^{-1/2} (LE^s - J3_h) \right\|^2}{NN_i + 2\sqrt{2NN_i}} \qquad \text{(EQ 9.46)}$$

## INPUT:

- $idec2_h$ - Index specifying requested deconvolution scenario for J2 and J3. (0 - no deconvolution, >0 - deconvolution scenario)
- *iflagdr* - Index specifying desired solution parameters. (-1 - no inversions, 0 - emission rates, visibilities, and wind phases, 1 - emission rates, 2 - amplitudes and phases)
- *NA* - Number of layers above inversion window.
- $\gamma_2$ and $\gamma_3$ - Weighting factors. (from CDB)

## HYPOTHESIS OF APPLICABILITY:

The volume emission rate profile is available for determining visibilities.

## CRITERIA OF APPLICABILITY:

Deconvolutions for visibility or amplitude, and wind phase profiles are requested and have been estimated to be possible. The length of the apparent quantities must be greater or equal to four. In addition, if the volume emission rate profile is available, the altitude of the volume emission rate maximum is inside the inversion window. ($idec2_h > 0$)

## TRANSFORMATION EQUATION:

(1) Call A3231 - Initialization of transformation equation arrays.

(2) Call A3232 - Eliminate lowest atmospheric layers with weak emission rates.

(3) In order to guarantee numerical stability of the deconvolution equations a lower limit of 1.0 is applied to the values for $\gamma_2$ and $\gamma_3$. (The labels $\gamma_2$ and $\gamma_3$ are used instead of $\gamma_{V^c}, \gamma_{A^c}$ and $\gamma_{22}$ (lower limit of 0.1 to $\gamma_{22} + \gamma_{33}$).) Also, the number *NA* of layers above the inversion window must be set equal to zero. The index $i_{add}$ for use by A3223 is also set equal to zero.

(4) Call A3233 - If requested, define the reference profiles for $V^c$. The specific input are *iref2* and *ind*=1.

(5) Call A3222 - If requested, define the constraint matrix $H_{V^c}$ or $H_{A^c}$. (with $i_{add}$=0)

If (*iflagdr* = 0) then

Input: $idec2_h$, *MAT2*, *NA*, $NN_i$, $x$, $\tilde{x}$, $i_{add}$ (=0), $\gamma_2$, $\tilde{L}$, $S2°$
Output: $H_{V^c}$

else

Input: $idec2_h$, *MAT2*, *MAT22* (or *MAT3*), *NA*, $NN_i$, $x$, $\tilde{x}$, $i_{add}$ (=0), $\gamma_2, \gamma_{22}$ (or $\gamma_E$), *L*, $S2°$
Output: $H_{A^c}$

end if

(6) Call A3223 - Solve for $V^c$ or $A^c$ using the Twomey matrix equation.

If ($iflagdr = 0$) then

Input: $idec2_h$, NA, $NN_i$, $\tilde{L}$, J2', $S2^\circ$, $\gamma_2$, $H_{V^c}$
Output: $idec2_h$, $V^c$

else

Input: $idec2h$, NA, $NN_i$, L, $J2^\circ$, $S2^\circ$, $\gamma_{22}$ (or $\gamma_E$) , $H_{A^c}$
Output: idec2h, A^c

end if

(7) Call A3225 - Derive the chi-square statistic ratio for $J2_h$.

If ($iflagdr = 0$) then

Input: $idec2_h$, $\tilde{L}$, J2',$S2^\circ$, NA, $NN_i$, $V^c$

else

Input: $idec2_h$, L, $J2^\circ$, $S2^\circ$, NA, $NN_i$, $A^c$

end if

Output: $chi2_h$

(8) Call A3233 - If requested, define the reference profiles for $V^s$. The specific input are $iref3$ and $ind = 2$.

(9) Call A3222 - If requested, define the constraint matrix $H_{V^s}$ or $H_{A^s}$. (with $i_{add}=0$)

If ($iflagdr = 0$) then

Input: $idec2_h$, MAT3, NA, $NN_i$, x, $\tilde{x}$, $i_{add}$ (=0), $\gamma_3$, $\tilde{L}$, $S3^\circ$
Output: $H_{V^s}$

else

Input: $idec2_h$, MAT33 (or MAT3), NA, $NN_i$, x, $\tilde{x}$, $i_{add}$ (=0), $\gamma_{33}$ (or $\gamma_3$) , L, $S3^\circ$
Output: $H_{A^s}$

end if

(10) Call A3223 - Solve for $V^s$ or $A^s$ using the Twomey matrix equation.

If ($iflagdr = 0$) then

Input: $idec2_h$, NA, $NN_i$, $\tilde{L}$, J3', $S3^\circ$, $\gamma_3$, $H_{V^s}$
Output: $idec2_h$, $V^s$

else

Input: $idec2_h$, $NA$, $NN_h$, $L$, $J3°$, $S3°$, $\gamma_{33}$ (or $\gamma_3$), $H_{A'}$
Output: $idec2h$, $A^s$

end if

(11) Call A3225 - Derive the chi-square statistic ratio for $J3_h$.

If ($iflagdr = 0$) then

Input: $idec2_h$, $L$, $J3°$, $S3°$, $NA$, $NN_h$, $V^s$

else

Input: $idec2_h$, $L$, $J3°$, $S3°$, $NA$, $NN_h$, $A^s$

end if

Output: $chi3_h$

(12) Call A3234 - Derive visibility, or amplitude, and wind phase profiles.

If ($iflagdr = 0$) then

Input: $idec2_h$, $iflagdr$, $V^c$, $V^s$, $NN_i$
Output: $V_h$, $\phi_h$

else if

Input: $idec2_h$, $iflagdr$, $A^c$, $A^s$, $NN_i$
Output: $A_h$, $\phi_h$

end if

(13) Call A3235 - Derive the standard deviation estimate arrays for the deconvolution solutions.

If ($iflagdr = 0$) then

Input: $iflagdr$, $idec2_h$, $V^c$, $V^s$, $S2°$, $S3°$, $L$, $NN_i$, $E°$, $\sigma_{E°}$
Output: $\sigma_{V_h}$, $\sigma_{\phi_h}$, $\sigma_{X_h}$, $\sigma_{X_h}$

else if

Input: $iflagdr$, $idec2_h$, $A^s$, $S2°$, $S3°$, $L$, $NN_i$
Output: $\sigma_{A_h}$, $\sigma_{\phi_h}$, $\sigma_{X_h}$, $\sigma_{X_h}$

end if

### 9.3.1   A3231: Initialize transformation equation arrays

**FUNCTIONAL DESCRIPTION:**

The apparent quantity arrays are adjusted to account for the volume emission rate solution of A322 and the volume emission rate profile is combined with the line-of-sight matrix.

**INPUT:**

- *iflagdr* - Index specifying desired solution parameters. (-1 - no inversions, 0 - emission rates, visibilities, and wind phases, 1 - emission rates, 2 - amplitudes and phases)
- $idec1_h$ - Index specifying if deconvolution scenario for $J1$ was successful. (0 - failed or no deconvolutions, >0 - deconvolution scenario.)
- $idec2_h$ - Index specifying deconvolution scenario for $J2$ and $J3$. (0 - no deconvolutions, 0 - deconvolution scenario.)
- $J1_i^o$, $J2_i^o$, and $J3_i^o$ - Apparent quantity arrays. (size: $NN_i$)
- $L_{i,j}$ - Line-of-sight matrix. (size: $NN_i$ x $(NN_i+NA)$)
- $NN_i$ - Dimension of arrays.
- $NA$ - Number of layers above inversion window.
- $E_i^o$ - Volume emission rate profile for combination of inversion window and layers added above window. (size: $NN_i + NA$)

**OUTPUT:**

- $NA$ - Number of layers above inversion window. Has been re-set as equal to zero.
- $idec2_h$ - Index specifying deconvolution scenario for $J2$ and $J3$. (0 - no deconvolutions, >0 deconvolution scenario)
- $J2_i'$, and $J3_i'$ - Modified apparent quantity arrays. (size: $NN_i$)
- $L_{i,j}$ - Line-of-sight matrix. (size: $NN_i$ x $NN_i$)
- $\tilde{L}_{i,j}$ - Line-of-sight matrix combined with the volume emission rate profile. (size: $NN_i$ x $NN_i$).

**HYPOTHESIS OF APPLICABILITY:**

None.

**CRITERIA OF APPLICABILITY:**

Deconvolutions are requested and the determination of the volume emission rate profile was attempted in A322. (i.e.: $idec2_h > 0$ and $iflagdr = 0$)

## TRANSFORMATION EQUATION:

(1) If the determination of the volume emission rate profile was not successful ( $FIXED\vec{J}_{\vec{k}l}$ ), then the visibility and wind phase profiles cannot be derived. Consequently, $idec2_h$ is re-set equal to zero. The following steps are done only if $idec2_h > 0$.

(2) The matrix is derived using

$$\tilde{L}_{i,j} = L_{i,j}E_j^o$$

(3) The primed apparent quantity arrays are derived using

$$\begin{bmatrix} J2_i' \\ J3_i' \end{bmatrix} = \frac{(LE^o)_i}{J1_i^o} \begin{bmatrix} J2_i^o \\ J3_i^o \end{bmatrix}$$

(4) If layers were added above the inversion window in A322, then these layers are combined by adding together the first $NA + 1$ columns of the matrices $L$ and $\tilde{L}$. The resulting matrices should be square and lower triangular with size $NN_i \times NN_i$.

(5) Following this matrix reduction, $NA$ is re-set as equal to zero.

### 9.3.2  A3232: Eliminate lowest atmospheric layers with weak emission rates

#### FUNCTIONAL DESCRIPTION:

In order to reduce the size of the solution equations and improve numerical stability for the deconvolutions, the lowest layers with weak volume emission rates are eliminated. If less than four layers result, then deconvolutions will not be done (i.e.: $idec2_h$ will be set equal to 0). The applied cutoff values for the emission rates are 5% of the maximum volume emission rate and one standard devation.

#### INPUT:

- *iflagdr* - Index specifying desired solution parameters. (-1 - no inversions, 0 - emission rates, visibilities, and wind phases, 1 - emission rates, 2 - amplitudes and phases)

- $idec2_h$ - Index specifying deconvolution scenario for J2 and J3. (0 - no deconvolutions, >0 - deconvolution scenario.)

- $J2_i^o$ and $J3_i^o$ - Apparent quantity arrays. (size: $NN_i$)

- $S2_i^o$ and $S3_i^o$ - Variance arrays for apparent quantity arrays. (size: $NN_i$)

- $E_{hi}$ - Volume emission rate profile for inversion window. (size: $NN_i$)

- $\sigma_{E_{hi}}$ - Standard deviation estimate array for $E_h$. (size: $NN_i$)

- $zcc_i$ - Altitude array. (km) (size: $NN_i$)

- $L_{i,j}$ - Line-of-sight matrix. (size: $NN_i$ x $NNi$)
- $NN_i$ - Current dimension of arrays.
- $J2_i'$ and $J3_i'$ - Modified apparent quantity arrays. (size: $NN_i$)
- - Line-of-sight matrix combined with the volume emission rate profile. (size: $NN_i$ x $NN_i$).

## OUTPUT:

- $idec2_h$ - Index specifying deconvolution scenario for $J2$ and $J3$. (0 - no deconvolutions, >0 deconvolution scenario)
- $NN_i$ - New dimension of arrays.
- $zcc_i$ - Reduced altitude array. (km) (size: $NN_i$)
- $E_i^o$ - Reduced volume emission rate array. (size: $NN_i$)
- $\sigma_{E_i^o}$ - Reduced standard deviation array for volume emission rates. (size: $NN_i$)
- $J2_i^o$ and $J3_i^o$ - Reduced apparent quantity arrays. (size: $NN_i$)
- $S2_i^o$ and $S3_i^o$ - Reduced apparent variance arrays.
- $L_{i,j}$ - Reduced line-of-sight matrix. (size: $NN_i$ x $NN_i$).
- $J2_i'$ and $J3_i'$ - Modified apparent quantity arrays. (size: $NN_i$)
- $\tilde{L}_{i,j}$ - Line-of-sight matrix combined with the volume emission rate profile. (size: $NN_i$ x $NN_i$).

## HYPOTHESIS OF APPLICABILITY:
None.

## CRITERIA OF APPLICABILITY:
Deconvolutions are requested and the volume emission rate profile has been derived in A322. (i.e.: $idec2_h > 0$ and $iflagdr = 0$)

## TRANSFORMATION EQUATION:

(1) The array reduction is based entirely on the volume emission rate profile $E_h$. First, the number $M$ of consecutive lowest layers which have a volume emission rate value $E_{hi}$ less the 5% of the maximum volume emission rate or less than one standard deviation is determined. All provided input arrays are reduced in size by eliminating elements

$NN_i$ - $M$ to $NN_i$. The resulting arrays are defined in the output list. The matrices $L$ and $\tilde{L}$, are reduced by eliminating the corresponding columns and rows. The final matrices should be square and lower triangular with dimension $(NN_i - M)$x$(NN_i - M)$.

(2) The final dimension $NN_i$ of the arrays is set by subtracting $M$ from the initial $NN_i$.

(3) If the final number of layers with volume emission rates $E_i^o$ larger than both 1.0 and $\sigma_{E_i^o}$ is less than four, then the deconvolutions will not be done, i.e. $idec2_h$ is set equal to zero.

### 9.3.3 A3233: Define reference profiles.

#### FUNCTIONAL DESCRIPTION:

Define a reference profile for $V^c$ or $A^c$, or for $V^s$ or $A^s$. The reference profile is defined using a selection index. At the moment only one choice is available. If requested, the reference profile will be used when defining the Twomey deconvolution equations.

#### INPUT:

- $idec2_h$ - Index specifying deconvolution scenario. (0 - no deconvolutions, >0 deconvolution scenario)
- $ind$ - Index specifying if reference profiles are to be produced for cosinusoidal (=1) or sinusoidal (=2) terms.
- $iref$ - (either iref2 or iref3) Index specifying selection of reference profile for $V^c$ or $A^c$ (when $ind$=1) or $V^s$ or $A^s$ (when $ind$=2)
- $NN_i$ - Dimension of arrays.
- $zcc_i$ - Altitude array. (km) (size: $NN_i$)
- $\Delta_\lambda$ - Optical path difference. ( $\mu m$) (see A333)
- $\Delta_\lambda^o$ - Corrected optical path difference. ($\mu m$) (not currently used)
- $\lambda_o$ or $N_F$ - Wavelength ($\mu m$) or filter number. (see A333)
- $iflagdr$ - Index specifying desired solution parameters. (-1 - no inversions, 0 - emission rates, visibilities and phases, 1 - emission rates, 2 - amplitudes and phases)
- $x_i$ - Reference profile for possible use in defining constraint matrix H. (may be available when $ind$=2) (size: $NN_i$)
- $J2_i^o$ and $J3_i^o$ - Reduced apparent quantity arrays (size: $NN_i$).
- $L_{i,j}$ - Reduced line of sight matrix (size $NN_i$ x $NN_i$).

#### OUTPUT:

- $x_i$ - Reference profile for possible use in defining constraint matrix $H$. (size: $NN_i$)

#### HYPOTHESIS OF APPLICABILITY:
None.

#### CRITERIA OF APPLICABILITY:
Twomey deconvolution is requested ($idec2_h > 0$).

### TRANSFORMATION EQUATION:

The reference profile $x$ is specified using the selection index *iref*. At the moment only one choice is available for these profiles. Therefore, for any *iref* value, the following is applied.

List of available choices

(1) If *iflagdr* equals 2, then the $x_i$ are set equal

$$\frac{\sqrt{\left(J2_i^o\right)^2 + \left(J3_i^o\right)^2}}{L_{i,i}} \geq 1.0$$

Otherwise, if (and only if) *ind*=1, the temperature profile from table A3233.1 is interpolated (e.g. linearly) to the altitude array *zcc*. For altitudes above 320 km and below 80 km, constant values will be sufficient. The temperatures are converted to visibilities using the relevant equation in A333. The array $x$ is set equal to the resulting visibility profile. If *ind* = 2, the input $x$ is unchanged and is passed again as output.

**Table A3233.1:**                    Reference temperature profile (average profile based on MSIS86* model.)

| Altitude (km) | Temperature (Kelvin) | Altitude (km) | Temperature (Kelvin) |
|---|---|---|---|
| 320 | 1132 | 155 | 837 |
| 300 | 1128 | 150 | 797 |
| 280 | 1123 | 145 | 750 |
| 260 | 1113 | 140 | 697 |
| 240 | 1099 | 135 | 636 |
| 230 | 1089 | 130 | 567 |
| 220 | 1076 | 125 | 486 |
| 210 | 1059 | 120 | 395 |
| 200 | 1038 | 115 | 294 |
| 195 | 1025 | 110 | 251 |
| 190 | 1011 | 105 | 200 |
| 185 | 994 | 100 | 186 |
| 180 | 976 | 95 | 184 |
| 175 | 955 | 90 | 197 |
| 170 | 931 | 85 | 225 |
| 165 | 903 | 80 | 256 |
| 160 | 873 | | |

\* - Reference: Hedin (1987).

### 9.3.4 A3234: Derive visibilities, or amplitudes, and wind phase profiles.

#### FUNCTIONAL DESCRIPTION:
Using the solutions derived from the Twomey deconvolutions, determine either the visibility and the wind phase profiles, or the amplitude and the wind phase profiles.

#### INPUT:
- $idec2_h$ - Index specifying deconvolution scenario. (0 - no deconvolutions, >0 deconvolution scenario)
- $iflagdr$ - Index specifying if visibilities (=0) or amplitudes (=2) are produced in A323.
- $V_i^c$ or $A_i^c$ - Visibility, or amplitude, cosinusoidal component profile (size: $NN_i$)
- $V_i^s$ or $A_i^s$ - Visibility, or amplitude, sinusoidal component profile. (size: $NN_i$)
- $NN_i$ - Dimension of arrays.

#### OUTPUT:
- $V_{hi}$ or $A_{hi}$ - Visibility profile (when $iflagdr = 0$) or amplitude profile (when $iflagdr = 2$). (size: $NN_i$)
- $\phi_{hi}$ - Wind phase component. (radians) (size: $NN_i$)

#### HYPOTHESIS OF APPLICABILITY:
None.

#### CRITERIA OF APPLICABILITY:
The deconvolutions were successful. ($idec2_h > 0$)

#### TRANSFORMATION EQUATION:

(1) When $iflagdr = 0$, the visibilities and phases $V_{hi}$ and $\phi_{hi}$ are determined using A3141 and A3142 (with $J1_h$, $J2_h$, and $J3_h$ replaced by 1, $V^c$, and $V^s$). When $iflagdr = 2$, the amplitudes $A_{hi}$ and the phases $\phi_{hi}$ are determined using A3142 and A3143.

(2) When $iflagdr = 0$, each visibility $V_{hi}$ greater than 0.99 or less than 0.1 should be assigned the value 1.0. The corresponding wind phase $\phi_{hi}$ is set equal to 0.0.

### 9.3.5 A3235: Derive standard deviation estimate arrays.

#### FUNCTIONAL DESCRIPTION:
Derive the standard deviation estimate arrays for either the visibilities and the wind phase profiles, or the amplitudes and the wind phase profiles.

## INPUT:

- *iflagdr* - Index specifying if visibilities (=0) or amplitudes (=2) are produced in A323.

- $idec2_h$ - Index specifying deconvolution approach. (not currently used)

- $V_i^c$ or $A_i^c$ - Visibility, or amplitude, cosinusoidal component profile (size: $NN_i$)

- $V_i^s$ or $A_i^s$ - Visibility, or amplitude, sinusoidal component profile. (size: $NN_i$)

- $S2_i^o$ and $S3_i^o$ - Variance arrays for the apparent quantity profiles.

- $L_{i,j}$ - Line-of-sight matrix (size: $NN_i$ x $NN_i$)

- $NN_i$ - Dimension of arrays.

- THE FOLLOWING IS NEEDED ONLY WHEN *iflagdr* = 0.

- $E_i^o$ - Volume emission rate profile. (size: $NN_i$)

- $\sigma_{E_i^o}$ - Standard deviation array for emission rate profile. (size: $NN_i$)

## OUTPUT:

- $\sigma_{V_{hi}}$ or $\sigma_{A_{hi}}$ - Visibility or amplitude error standard deviation estimate arrays. (size: $NN_i$)

- $\sigma_{\phi_{hi}}$ - Phase error standard deviation estimate array. (size: $NN_i$)

- $\sigma_{X_{hi}^c}^2$ and $\sigma_{X_{hi}^s}^2$ - Intermediate error standard deviation arrays. (size: $NN_i$)

## HYPOTHESIS OF APPLICABILITY:
None.

## CRITERIA OF APPLICABILITY:
The deconvolutions were successful. ($idec2_h > 0$)

## TRANSFORMATION EQUATION:

(1) Define the direct inversion standard deviation arrays for the visibility, or amplitude, cosinusoidal and sinusoidal components.

$$\sigma_{x_i^c}^2 = \left( L^T S_{J2_h}^{-1} L \right)_{i,i}^{-1}$$

where $S_{J2_h}$ is diagonal with $S_{J2_{hi,i}} = S2_i^o$, and

$$\sigma_{x_i^s}^2 = \left( L^T S_{J3_h}^{-1} L \right)_{i,i}^{-1}$$

where $S_{J3_h}$ is diagonal with $S_{J3_{hi,i}} = S3_i^o$. (see A3226)

(2) If *iflagdr* = 2, then set $\sigma^2_{X_h^c} = \sigma^2_{x_c}$ and $\sigma^2_{X_h^s} = \sigma^2_{x_s}$. Otherwise (i.e.: *iflagdr* = 0), set

$$\sigma^2_{X_h^c} = \frac{\left[\sigma^2_{x_i^c} - (V_i^c)^2 \sigma^2_{E_i^o}\right]}{(E_i^o)^2 + \sigma^2_{E_i^o}}$$

and

$$\sigma^2_{X_h^s} = \frac{\left[\sigma^2_{x_i^s} - (V_i^s)^2 \sigma^2_{E_i^o}\right]}{(E_i^o)^2 + \sigma^2_{E_i^o}}$$

A lower limit of $\sigma^2_{E_i^o}$ should be applied to the terms denoted by the square brackets. The above two equations stem from the variance expression of the product of two uncorrelated variates, i.e.

$$\sigma^2_{ab} = \sigma^2_a \sigma^2_b + \sigma^2_a \mu^2_b + \sigma^2_b \mu^2_a$$

where $a$ and $b$ are the uncorrelated variates, and $\mu_a$ and $\mu_b$ are their respective means.

(3) The phase standard deviation estimates are given by

$$\sigma_{\phi_{hi}} = \frac{\sqrt{\sigma^2_{X_{hi}^s} (V_i^c)^2 + \sigma^2_{X_{hi}^c} (V_i^s)^2}}{(V_i^c)^2 + (V_i^s)^2}$$

with $V^c$ and $V^s$ replaced by $A^c$ and $A^s$ when *iflagdr* = 2. When *iflagdr* = 0, the visibility standard deviation estimates are specified by

$$\sigma_{V_{hi}} = \frac{\sqrt{\sigma^2_{X_{hi}^c} (V_i^c)^2 + \sigma^2_{X_{hi}^s} (V_i^s)^2}}{(V_i^c)^2 + (V_i^s)^2}$$

and, when *iflagdr* = 2, the amplitude standard deviation estimates are defined by

$$\sigma_{A_{hi}} = \frac{1}{2} \frac{\sqrt{\sigma^2_{X^c_{hi}} (A^c_i)^2 + \sigma^2_{X^s_{hi}} (A^s_i)^2}}{(A^c_i)^2 + (A^s_i)^2}$$

(4) When *iflagdr* = 0, for each visibility $V_{hi}$ greater that 0.99 or less than 0.1, both the corresponding visibility and wind phase standard deviation estimates should be set equal to $2 \times 10^{30}$ in the CDB (or some other large number).

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Twomey, S. (1963), 'On the numerical solution of Fredholm integral equations of the first kind by the inversion of the linear systems produced by quadrature', J. Assoc. Comput. Mach., Vol. 10, pp. 97-101.

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# CHAPTER 12

# A42 - Select data with respect to latitude

## 12.1  A421: Select Data with Respect to Latitude

**FUNCTIONAL DESCRIPTION:**

Meridional wind, zonal wind and temperature profiles are selected for the 3AL grid corresponding to 4 degree intervals of latitude.

**INPUT:**

The input parameters are:

- $\vartheta_{grid}$ - 3AL grid point latitude.

- $O\vartheta_p$ - The latitude of the measurements found in the FOV combination A343. This is an array of the individual latitudes found in A343.

- *OTflag* - Overlap temperature flag.

- *OEflag* - Overlap emission flag.

- $\Delta_{lat}$ - Change in latitude.

**OUTPUT:**

The output parameters are:

- $(\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})_{grid}$ - latitudes of data points on each side of the grid latitude.

- $n_s$ - number of profiles selected.

**HYPOTHESIS OF APPLICABILITY:**

It is assumed that linear interpolation in latitude is adequate. It is assumed that the sets $(\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})$ will be found for the full day's data.

### CRITERIA OF APPLICABILITY:

If no data exist within 2 degrees of the grid point, the value is left empty and no latitude set is generated. If both OH rotational temperatures and doppler temperatures are available for a given measurement then use the rotational temperatures for the gridding. Select only deconvolved data (see A41).

### TRANSFORMATION EQUATION:

Select data within $\Delta_{lat}$ degrees of the grid latitude:

$$\text{If } (\vartheta_{grid} - \Delta_{lat}) \leq O\vartheta_p \leq (\vartheta_{grid} + \Delta_{lat}) \text{ then}$$
$$\text{If } OTflag_p = 1 \text{ and } OEflag_p = 1 \text{ then}$$
$$O\vartheta_p \in (\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})$$
$$n_s = n_s + 1$$

(EQ 12.1)

# CHAPTER 13

# A42 - Select data with respect to latitude

## 13.1 A421: Select Data with Respect to Latitude

### FUNCTIONAL DESCRIPTION:
Meridional wind, zonal wind and temperature profiles are selected for the 3AL grid corresponding to 4 degree intervals of latitude.

### INPUT:
The input parameters are:

- $\vartheta_{grid}$ - 3AL grid point latitude.

- $O\vartheta_p$ - The latitude of the measurements found in the FOV combination A343. This is an array of the individual latitudes found in A343.

- $OTflag$ - Overlap temperature flag.

- $OEflag$ - Overlap emission flag.

- $\Delta_{lat}$ - Change in latitude.

### OUTPUT:
The output parameters are:

- $(\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})_{grid}$ - latitudes of data points on each side of the grid latitude.

- $n_s$ - number of profiles selected.

### HYPOTHESIS OF APPLICABILITY:
It is assumed that linear interpolation in latitude is adequate. It is assumed that the sets $(\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})$ will be found for the full day's data.

## CRITERIA OF APPLICABILITY:

If no data exist within 2 degrees of the grid point, the value is left empty and no latitude set is generated. If both OH rotational temperatures and doppler temperatures are available for a given measurement then use the rotational temperatures for the gridding. Select only deconvolved data (see A41).

## TRANSFORMATION EQUATION:

Select data within $\Delta_{lat}$ degrees of the grid latitude:

$$\text{If } (\vartheta_{grid} - \Delta_{lat}) \leq O\vartheta_p \leq (\vartheta_{grid} + \Delta_{lat}) \text{ then}$$
$$\text{If } OTflag_p = 1 \text{ and } OEflag_p = 1 \text{ then}$$
$$O\vartheta_p \in (\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})$$
$$n_s = n_s + 1$$

(EQ 13.1)

# CHAPTER 14

# A42 - Select data with respect to latitude

## 14.1 A421: Select Data with Respect to Latitude

### FUNCTIONAL DESCRIPTION:

Meridional wind, zonal wind and temperature profiles are selected for the 3AL grid corresponding to 4 degree intervals of latitude.

### INPUT:

The input parameters are:

- $\vartheta_{grid}$ - 3AL grid point latitude.

- $O\vartheta_p$ - The latitude of the measurements found in the FOV combination A343. This is an array of the individual latitudes found in A343.

- $OTflag$ - Overlap temperature flag.

- $OEflag$ - Overlap emission flag.

- $\Delta_{lat}$ - Change in latitude.

### OUTPUT:

The output parameters are:

- $(\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})_{grid}$ - latitudes of data points on each side of the grid latitude.

- $n_s$ - number of profiles selected.

### HYPOTHESIS OF APPLICABILITY:

It is assumed that linear interpolation in latitude is adequate. It is assumed that the sets $(\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})$ will be found for the full day's data.

### CRITERIA OF APPLICABILITY:

If no data exist within 2 degrees of the grid point, the value is left empty and no latitude set is generated. If both OH rotational temperatures and doppler temperatures are available for a given measurement then use the rotational temperatures for the gridding. Select only deconvolved data (see A41).

### TRANSFORMATION EQUATION:

Select data within $\Delta_{lat}$ degrees of the grid latitude:

$$\text{If } (\vartheta_{grid} - \Delta_{lat}) \leq O\vartheta_p \leq (\vartheta_{grid} + \Delta_{lat}) \text{ then}$$
$$\text{If } OTflag_p = 1 \text{ and } OEflag_p = 1 \text{ then}$$
$$O\vartheta_p \in (\vartheta_1, \vartheta_2, ..., \vartheta_{n_s})$$
$$n_s = n_s + 1$$

(EQ 14.1)